COMPUTER-AIDED CIVIL AND INFRASTRUCTURE ENGINEERING

Computer-Aided Civil and Infrastructure Engineering **33** (2018) 282–299

Two Methods to Calibrate the Total Travel Demand and Variability for a Regional Traffic Network

Tao Wen*

School of Civil and Environmental Engineering, University of New South Wales, Sydney, Australia and Data61, CSIRO, Sydney, Australia

Lauren Gardner, Vinayak Dixit & S. Travis Waller

School of Civil and Environmental Engineering, University of New South Wales, Sydney, Australia

Chen Cai & Fang Chen

Data61, CSIRO, Sydney, Australia

Abstract: *This article proposes a novel methodology that uses the bi-level programming formulation to calibrate the expected total demand and the corresponding demand variability of traffic networks. In the bi-level formulation the upper-level is either a new maximum likelihood estimation method or a least squares method and the lower-level is the strategic user equilibrium assignment model (StrUE) which accounts for the day-to-day demand volatility. The maximum likelihood method proposed in this article has the ability to utilize information from day-to-day observed link flows to provide a unique estimation of the total demand distribution, whereas the least squares method is capable of capturing link flow variations. The lower-level StrUE model can take the total demand distribution as input, and output a set of link flow distributions which can then be compared to the link-level observations. The mathematical proof demonstrates the convexity of the model, and the sensitivity to the prediction error is analytically derived. Numerical analysis is conducted to illustrate the efficiency and sensitivity of the proposed model. Some possible future research is discussed in the conclusion.*

1 INTRODUCTION

Enhanced origin–destination (O–D) matrix estimation methodologies could prove useful for transportation planning. Traditionally, the O–D matrix is obtained by trip generation and distribution module using data from plate surveys, household surveys, or roadside surveys (Castillo et al., 2014). Such survey activities may suffer from limited response and sampling coverage. As an alternative, there are some statistical approaches for estimating or calibrating an O–D matrix from observed link flows and some prior knowledge of the O–D demand. However, it is difficult to infer a unique O–D matrix directly using these approaches because the number of O–D pairs is much larger than the number of links; thus statistical assumptions or some prior information on O–D matrix are necessary to guarantee a unique solution. For example, prior O–D matrix may be used as a regularization which ensures the objective function to be a convex one (Menon et al., 2015). Unlike the traditional O–D estimation, this article proposes a supplementary approach for calibrating the aggregated O–D demand and corresponding variability for a regional traffic network using day-to-day observed link counts, assuming that the demand proportions are known *a priori* information. The proposed method explicitly considers the inherent volatility in demand which is commonly observed in traffic networks, in conjunction with the link flow fluctuation "caused" by demand volatility.

In this article, we present two methods to calibrate the total demand distribution. Both methods can be represented as a bi-level programming model, which differ only in the upper-level problem. The lower-level problem for both methods is the strategic user equilibrium assignment model (StrUE), which accounts for day-to-day demand volatility. The upper-level model is either

*To whom correspondence should be addressed. E-mail: *tao.wen@data61.csiro.au*.

DOI: 10.1111/mice.12278

a maximum likelihood estimation method or the least squares method. Each of the two calibration methods is defined as follows:

(1) The maximum likelihood method for O–D matrix estimation with StrUE which is hereby referred to as the MLOD method;
(2) The least squares method for O–D matrix estimation with StrUE hereby referred to as LSOD.

The novelty of both methods evolves from the incorporation of the strategic user equilibrium model (referred to as StrUE in this article) (Dixit et al., 2013). The StrUE model is defined such that "at strategic user equilibrium all used paths have equal and minimal expected cost." For each user present in the network, they make a route choice decision based on their knowledge of demand distribution, then this chosen route is followed regardless of the realized travel demand on a given day (i.e., given demand scenario). Therefore, under StrUE, the link flows will not necessarily result in an equilibrium state for any particular demand realization; instead, equilibrium exists stochastically over all demand realizations. The StrUE model was proposed to capture the impact of day-to-day demand volatility on network performance, and eventually route choice within the static user equilibrium framework. The StrUE model can take the total demand distribution as input (estimated at the upper level), and output a set of link flow distributions, which can then be compared to the link-level observations.

In the StrUE model, each O–D pair demand is assumed to be a fixed proportion of the total demand in the system; hence each O–D pair demand varies according to the change in total demand. Therefore, the objective of this work is to estimate the total demand, and perhaps more importantly, the parameters which define the total trip demand distribution. Given the estimated demand distribution, the proposed method also provides the variance of link flows (from StrUE), which can be used as a measurement of reliability for planning purposes.

The bi-level programming method is proposed to eliminate the impact of strongly biased prior estimates, where the upper-level provides information about the total demand distribution to the lower-level StrUE model, and the results from the StrUE model can provide link flow distributions back to the upper level. A benefit of the proposed model includes the incorporation of observed day-to-day link flows, instead of aggregated or averaged values. Additionally, the performance of both MLOD and LSOD methods can be assessed by comparing the estimated link flow distributions (which are a direct output of the StrUE model based on the estimated total demand distribution) with the simulated day-to-day link flows.

The association of link flow variables to the total demand in StrUE allows for the use of day-to-day link flows (which in return provide actual link flow distributions) to calibrate the total demand distribution. The calibration is accomplished by implementing the following methods: (1) MLOD method, in which we maximize the joint probability of observing the entire link flows within a time period and (2) LSOD method, in which we minimize the sum of the residuals of mean and standard deviation of link flows. The main difference between the two methods is that the MLOD method considers every observation of link flow, and seeks to find a distribution that fits the observed link flow best, whereas the LSOD method uses only the mean and standard deviation of link flow as the inputs. The LSOD finds the O–D demand that can produce a mean and standard deviation of link flows similar to the observed ones. When data sets are typically small or moderate in size, extensive simulation studies show that in small sample designs where there are only a few failures or errors, the maximum likelihood estimation is better than the least squares method (Maus et al., 2001; Genschel and Meeker, 2010).

An additional contribution of this research addresses the issue of low coverage rate or failure of loop detectors, which can lead to error in observed link flows (Zhou and List, 2010). One way to mitigate the low coverage rate issue was investigated by various sensor location problem models (Gan et al., 2005; Ehlert et al., 2006; Yang et al., 2006; Larsson et al., 2010; Gentili and Mirchandani, 2012). In this work, sensitivity analysis is conducted to demonstrate the model's robustness against varying levels of detector error. In addition, the sensitivity to error in observed link flows is analytically derived for the proposed model, and the results are validated using simulation.

The remainder of this article includes a literature review of previous research in Section 2. Section 3 defines the mathematical model and includes a derivation for the analytical solution to the total demand estimation. Numerical analysis is demonstrated in Section 4 where the estimated results of both methods are compared; conclusions, limitations of the model, and future research are presented in Section 5.

2 LITERATURE REVIEW

Historically, O–D matrix estimation and its calibration mainly relied upon statistical approaches using loop count data. O–D estimation research has since been expanded to include a wide range of methods such as the generalized least square method (Cascetta, 1984; Bell, 1991), the maximum likelihood (Spiess, 1987), bi-level programming approach (Yang et al., 1992), Bayesian approaches (Maher, 1983; Tebaldi and West, 1998), and maximum entropy (Fisk, 1988). Integration of the methods mentioned above was also of recent interest (Aerde et al., 2003; Castillo et al., 2014). Statistically, the maximum likelihood method estimates a set of parameters for a probability density function to fit the observations, that is, it maximizes the joint probability of observing the existing data, and a preassumed distribution is normally required. The method of least squares is a standard approach to approximate the solution of overdetermined systems, that is, sets of equations in which there are more equations than unknowns, such as linear regression. In this article, these two traditional methods, generalized least square method and maximum likelihood, are modified and applied to calibrate the total network demand.

In many literatures, estimation of the O–D matrix is a once-off procedure, that is, calibration/estimation of O–D matrix is only performed once. This may lead to some issues when the network is congested, the prior information on O–D matrix is inaccurate, or data noise is not insignificant. In the bi-level programming approach proposed in this article, the upper level is an O–D matrix estimation problem and the lower level is the assignment of O–D trips. Yang et al. (1992) first introduced the convex bi-level optimization problem, which was later extended to account for link flow correlation (Yang, 1995). The heuristic algorithm, which is a global optimum technique, was applied to solve the upper level in Yang (1995), Kim et al. (2001), and Stathopoulos and Tsekeris (2004); however, the solution was not proved to be optimal mathematically. Codina et al. (2006) proposed two algorithms under the bi-level programming framework: one sought for an approximation of the steepest descent direction for the upper level and one linearized the lower-level assignment problem. In addition, an iterative column generation algorithm was demonstrated based on the characteristics of path cost function continuity, and the solution was proved to be a local minimum (Garcia-Rodenas and Verastegui-Rayo, 2008). However, the aforementioned algorithms were only applied on a small network. In this article the proposed bi-level programming approach is proved to be applicable to medium-sized networks such as the Anaheim network.

Typically, the objective of O–D matrix estimation is to optimize an objective function (which may vary based on model requirements) subject to a set of constraints (typically the flow conservation and the link-path incidence relationship). However, the problem is often challenging due to that the number of monitored links in a traffic network is often much smaller than the number of O–D pairs to be estimated; therefore it may not be possible to obtain a unique solution from a single set of link counts alone. Furthermore, the problem has been extended to account for the stochastic nature of flows (Lo et al., 1996; Lo et al., 1999), and the time-dependent characteristics of the network (Bierlaire and Crittin, 2004; Frederix et al., 2011). Some computer-aided heuristic algorithms are also applied to this problem (Stathopoulos and Tsekeris, 2004). The genetic algorithm, which is a heuristic search method, plays an important role in the O–D estimation problem (Kim et al., 2001; Baek et al., 2004; Yun and Park, 2005; Kattan and Abdulhai, 2006). The main advantage of the genetic algorithm is its capability of solving nonconvex, complex optimization, whereas the drawback is that the solution is not guaranteed to be optimal. In addition, path flow estimator techniques also provide O–D specific flows, which could be used in the estimation of O–D matrix (Sherali et al., 1994; Chootinan et al., 2005; Chen et al., 2009) but this model either needs path enumeration (Sherali et al., 2003) or information on the set of shortest paths (Nie and Lee, 2002; Nie et al., 2005). Some other methods have also been proposed by researchers to enhance the model applicability, such as multi-class O–D estimation (Baek et al., 2004; Wong et al., 2005), fuzzy-based approach (Xu and Chan, 1993; Reddy and Chakroborty, 1998; Foulds et al., 2013) and neutral network-based approach (Gong, 1998). However, issues regarding computation complexity and the application to large-scale networks still remain a challenge.

On the other hand, higher order information of a network, such as the variance and covariance of observed link flows, can potentially provide more constraints to the optimization problem. This is considered as network tomography problems in statistics and computer science literature (Vardi, 1996; Cao et al., 2000; Airoldi and Blocker, 2013; Hazelton, 2015), but its application in transportation models is yet to be fully explored. Cremer and Keller demonstrated that aggregating or averaging link count data collected over a sequence of time periods may result in the loss of important information (Cremer and Keller, 1987). Hazelton (2003) proposed a weighted least squares method to account for the covariance of links and assumed a parameter to explain the circumstances when the variance exceeds the mean if a Poisson distribution is used. Bell (1983) proposed a maximum likelihood method and found the analytical solution to the covariance of O–D matrix by using a Taylor approximation. However, the assumptions made in these studies may limit the model applicability. For example, the O–D demand was assumed to follow the Poisson or multinomial distribution, which stipulates certain relationships between the mean and variance of the O–D demand. In monitored networks, loop detectors can easily provide link counts on a day-to-day basis; therefore, it is important to consider the variation of link flows and the distribution of O–D total demand as effective information to calibrate the O–D trip matrix (Wen et al., 2015). In this study, the StrUE traffic assignment model assumes the O–D demand follows a lognormal distribution, which allows the mean and variance of total demand to be different from each other, and assures the nonnegativity of the demand. The proposed model in this article, therefore, uses link flow variation to provide a robust estimation of total demand while maintaining computation efficiency.

Estimation of the O–D trip matrix also requires a proper assignment model. When applying the assignment model to a large network, realism and computational complexity are both critical in determining a model's practical applicability. Further, a major complication in transportation modeling is the ability to properly account for the inherent uncertainties regarding demand (Bellei et al., 2006; Kim et al., 2009; Szeto et al., 2011) and capacity levels (Brilon et al., 2005; Wu et al., 2010). Additionally, as has been noted, uncertainty regarding these variables directly affects route choice behavior (Uchida and Iida, 1993) and traffic predictions (Duthie et al., 2011). It is, therefore, necessary to incorporate these stochastic elements into models to ensure robust planning capabilities, but to do so in a manner that maintains computational tractability. The strategic user equilibrium (Dixit et al., 2013) effectively accounts for the impact of demand uncertainty subject to Wardrop's UE conditions, and under the static user equilibrium, the computation tractability and simplicity are preserved. The model was extended to the dynamic traffic assignment (Waller et al., 2013), road pricing scheme (Duell et al., 2014), and independently distributed O–D demands (Wen et al., 2016).

The contribution of this study can be summarized as follows:

(1) We apply the strategic user equilibrium model, which explicitly accounts for demand volatility in users' routing mechanism. It can also provide link flow fluctuation based on demand volatility.
(2) We assume the total demand follows a lognormal distribution, a lognormal distribution allows the variance and the mean of the total demand to be independent, and nonnegative demand estimation is guaranteed.
(3) Given the day-to-day link flow fluctuations, we estimate the total network travel demand distribution using two different methods: (i) the MLOD method and (ii) LSOD method. The performance of both methods is evaluated and compared for a medium-sized network and large-sized network.
(4) A bi-level formulation is proposed, which reduces the impact of biased initial estimates. Both upper level and lower level are proven to be strictly convex.
(5) O–D estimation results from the proposed methods are presented and compared for both analytical and simulated analysis.

3 PROBLEM FORMULATION

This section defines the mathematical concept of the model. Table 1 defines the notations used in this article.

In this article, two assumptions are made to guarantee consistency, uniqueness, and computation simplicity:

(1) Each O–D pair demand is proportional to the total demand and the demand proportions are fixed. That is, the objective of this article is to scale each O–D demand distribution while preserving the demand proportions. Statistically, this means we have more confidence in the demand proportions but not the exact number of trips. Similar to the traditional O–D estimation approaches where even a full prior O–D matrix is assumed to be known (not necessarily, but in several models the prior O–D matrix guarantees uniqueness of the solutions), the demand proportions can be regarded as prior information, which is obtained from other transportation techniques such as gravity model or census data. These techniques can provide comparatively reliable demand proportions, and the use of loop detector data in this article provides a way to calibrate the number of trips. Note that this assumption of known *priori* O–D proportions may limit the model's applicability, but it also guarantees the model's uniqueness. In some scenarios, such an assumption may be relaxed if uniqueness is not considered critical.
(2) The trip demand is assumed to follow a certain statistical distribution; previously a lognormal distribution has been used (Kamath and Pakkala, 2002; Zhao and Kockelman, 2002; Duell et al., 2014; Wen et al., 2014). Under the assumption of a log-normally distributed demand, this article focuses on estimating the demand distribution parameters. Note that other distributions can also be used if they do not change the convexity of the objective function, and preserve the positiveness. In addition, for the StrUE model a log-normal distributed total trip demand allows for a closed form

Table 1
Summary of notations

Symbol	*Definition*
N	Link (index) set.
K_{rs}	Path set for O–D pair rs.
A	Node (index) set.
p_n	Proportion of total demand on link n; $P = (p_1, \ldots, p_n)$.
$t_n()$	Travel time on link n; $\boldsymbol{t} = (\ldots, t_n, \ldots)$.
t_{nf}	Free flow travel time on link n.
q_{rs}	Fraction of total demand that are between O–D pair r-s; $\sum_{\forall rs} q_{rs} = 1$.
T	Random variable for total demand with probability distribution $g(T)$.
$g(T)$	Lognormal probability density function of the total demand.
x_{ni}	Observed flow on link n, for day i.
l_n	Flow on link n.
h_k^{rs}	The proportion of flow on path k, connecting O–D pair r-s.
C_n	The capacity on link n.
$\delta_{n,k}^{m}$	Indicator variable $\delta_{n,k}^{rs} = \{\begin{matrix} 1\ if\ n\ is\ included\ in\ path\ k \\ 0\ otherwise \end{matrix}$, which indicates if link n is on path k for O–D pair rs. $(\Delta^{rs})_{n,k} = \delta_{n,k}^{rs}; \Delta = (\ldots, \Delta^{rs}, \ldots)$
μ	The location parameter for a lognormal distribution, which is also mean of the corresponding normal distribution.
σ^2	The scale parameter for a lognormal distribution, which is also the variance of the corresponding normal distribution.
m_T	The expected total demand.
v_T	The variance of the lognormal distribution.
m_n	The expected link flow on link n.
v_n	The variance of link flow on link n.
s_T	The standard deviation of the total demand.
s_n	The standard deviation of flow on link n.
cov	The coefficient of variation, which is defined as the ratio of the standard deviation to the mean of a variable.
Std()	The standard deviation of a variable.
$E()$	The mean/expectation of a variable.

expression of the probability density function, which helps to construct the model analytically.

3.1 The Strategic user equilibrium assignment model (StrUE)

The StrUE model is used as the assignment model for the O–D estimation, and details of the StrUE model can be found in Dixit et al. (2013). In summary, the StrUE model has the following objective function:

$$\text{Minimize} : z([p]) = \sum_{n \in N} \int_0^{p_n} \int_{-\infty}^{+\infty} t_n(fT)\ g(T)\, dT\, df, \quad (1)$$

Subject to:

$$\sum_k h_k^{rs} = q_{rs}, \qquad \forall k, r, s, \quad (2)$$

$$h_k^{rs} \geq 0, \qquad \forall k, r, s, \quad (3)$$

$$p_n = \sum_r \sum_s \sum_k h_k^{rs} \delta_{n,k}^{rs}, \qquad \forall k, r, s. \quad (4)$$

In this formulation, users' strategic choice is denoted as the link proportions p_n, that is, the link flow divided by the total demand T; $[p]$ represents a vector of all the link proportions; and f is a dummy variable for integration. Each O–D demand is the fixed demand proportion q_{rs} multiplied by the total demand T, that is, each O–D demand is proportional to the total demand, and the proportions are fixed constants. This implies that all O–D demands are perfectly correlated. The total demand is assumed to follow a certain distribution. The objective function is the sum of the integrals of the expected value of the link cost functions; the link proportions are proved to be unique under the assumption of fixed demand proportions. Note that in StrUE users will stick to their link choice proportions p_n regardless of the realized total demand day-to-day (i.e., once decided, p_n is a constant in the formulation), whereas link flow does vary every day corresponding to the realized total demand. In many O–D demand calibration literatures, the users' route choice is assumed to be either a known *a priori* or obtained from a traditional assignment model. In this article, the StrUE explicitly accounts for demand uncertainty and its impact on users' route choice while maintaining the computation simplicity, and is integrated into the total demand calibration context, which has rarely been done in the past literature.

The link travel time function for the StrUE model is defined by the U.S. Bureau of Public Roads (U.S., 1964) cost function due to its widespread use in transport planning models:

$$t_n\ (l_n) = t_{nf} \left[1 + \alpha \left(\frac{l_n}{C_n}\right)^{\beta}\right], \quad (5)$$

where α and β are the parameters for the BPR function. The fraction of the total demand between O–D pair r-s, namely q_{rs}, can be obtained from the prior estimates, that is, census data, gravity model. The total demand is assigned to each link by the link proportions:

$$l_n = p_n\ T\ \ p_n \in P, n \in N. \quad (6)$$

The link proportions are obtained from the StrUE assignment model, therefore it is known constant in the O–D matrix estimation problem and total demand is the stochastic variable; from Equation (6), each link flow is the link proportions multiplied by the total demand variable. According to the properties of lognormal distribution, if the total demand variable follows a lognormal distribution, then the link flow represented in Equation (6) also follows a lognormal distribution, which is related to the total demand distribution by the following equations:

$$m_n = p_n\, m_T, \tag{7}$$

$$v_n = p_n^2\, v_T. \tag{8}$$

3.2 The maximum likelihood O–D estimation incorporating the strategic user equilibrium (MLOD)

In MLOD, we first find the parametric expression of the probability density function of link flow distribution. The parameters for the link flow distribution can be obtained by the definition of lognormal distribution:

$$\mu_n = \ln m_n - \frac{1}{2}\ln\left(1 + \frac{v_n}{m_n^2}\right), \tag{9}$$

$$\sigma_n^2 = \ln\left(1 + \frac{v_n}{m_n^2}\right). \tag{10}$$

Substitute Equations (7) and (8) into Equations (9) and (10), we have the transformation of the total demand distribution to link flow distribution:

$$\sigma_n = \sigma_T, \tag{11}$$

$$\mu_n = \ln p_n + \mu_T. \tag{12}$$

It is important to realize that μ and σ^2, which appear in the equations of the lognormal distribution, do not denote the mean and the variance of the lognormal distribution, but of the corresponding parameters of the normal distribution. The mean and the variance of the lognormal distribution are indicated in the following discussion by m and v. Since each link flow follows a lognormal distribution, the probability density function of observing x_{ni} trips on link n is

$$P\ (x_{ni}) = \frac{1}{x_{ni}\sigma_n\sqrt{2\pi}}\, e^{-\frac{(\ln x_{ni} - \mu_n)^2}{2\sigma_n^2}} \quad n \in N, \tag{13}$$

where, x_{ni} is the observed flow on link n for a fixed time period (e.g., the flow rate during morning peak hours) on day i. Here the observed link flows are indicated by an n-by-i matrix, where n is the number of links and i is the number of observations:

$$X_{ni} = \begin{bmatrix} x_{11} & \cdots & x_{1i} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{ni} \end{bmatrix}. \tag{14}$$

The maximum likelihood method here is to maximize the joint probability of observing all sets of link flows, in order to reduce the effect of noise and observation failure. The joint probability density function is given by the following equation:

$$j\,(X_{ni}) = \prod_1^i \prod_1^n \frac{1}{x_{ni}\sigma_n\sqrt{2\pi}}\, e^{-\frac{(\ln x_{ni} - \mu_n)^2}{2\sigma_n^2}}. \tag{15}$$

Conventionally, we maximize the logarithm of the joint probability, because taking the log of the function will not change its convexity. By plugging Equations (11) and (12) into Equation (15) and changing the signs, the objective function becomes:

$$\min:\ J\,(\mu_T, \sigma_T) = \sum_1^i \sum_1^n \left[\ln(x_{ni}\sigma_T\sqrt{2\pi}) + \frac{\left(\ln\frac{x_{ni}}{p_n} - \mu_T\right)^2}{2\sigma_T{}^2}, \right. \tag{16}$$

subject to: $\sigma_T > 0$.

The Hessian matrix of the objective function is positive definite, hence the function is strictly convex, which guarantees the solution is globally optimal and unique. Note that convexity is critical for transport planners, because if there exist multiple optimal solutions to the objective function it would be difficult to perform the subsequent analysis which is based on the estimated O–D matrix. The optimal solutions can be found by taking the first derivative with respect to mean and variance of total demand:

$$\mu_T = \frac{\sum_1^i \sum_1^n \ln\frac{x_{ni}}{p_n}}{ni}, \tag{17}$$

$$\sigma_T^2 = \frac{\sum_1^i \sum_1^n \left(\ln\frac{x_{ni}}{p_n} - \mu_T\right)^2}{ni} \tag{18}$$

Sensitivity is a measurement of a model's robustness, in the proposed model, observed loop counts may be erroneous due to loop detector failure, and here the error term is expressed as a matrix that has the same dimension as the observed loop counts:

$$E_{ni} = \begin{bmatrix} e_{11} & \cdots & e_{1i} \\ \vdots & \ddots & \vdots \\ e_{n1} & \cdots & e_{ni} \end{bmatrix}. \tag{19}$$

Note that in lots of previous literature the error term is assumed to be Gaussian, that is, the error term is a variable which follows a normal distribution. More generally, in this article the error term is modelled as a variable, and thus no distributional assumptions are required. The loop count matrix with error term is:

$$R_{ni} = X_{ni} + E_{ni} = \begin{bmatrix} x_{11}+e_{11} & \cdots & x_{1i}+e_{1i} \\ \vdots & \ddots & \vdots \\ x_{n1}+e_{n1} & \cdots & x_{ni}+e_{ni} \end{bmatrix}. \quad (20)$$

The corresponding estimated μ_T and σ_T^2 based on observed loop counts with error term are represented as $R\mu_T$ and $R\sigma_T^2$:

$$R\,\mu_T = \frac{\sum_1^i \sum_1^n \ln \frac{x_{ni}+e_{ni}}{p_n}}{ni}, \quad (21)$$

$$R\,\sigma_T^2 = \frac{\sum_1^i \sum_1^n \left(\ln \frac{x_{ni}+e_{ni}}{p_n} - R\mu_T\right)^2}{ni}. \quad (22)$$

From the definition of lognormal distribution, the corresponding expected total demand with and without error term are expressed as Rm_T and m_T, respectively:

$$m_T = e^{\mu_T+0.5\sigma_T^2} = e^{\frac{\sum_1^i \sum_1^n \ln \frac{x_{ni}}{p_n} + 0.5 \sum_1^i \sum_1^n \left(\ln \frac{x_{ni}}{p_n} - \mu_T\right)^2}{ni}}, \quad (23)$$

$$Rm_T = e^{R\mu_T+0.5R\sigma_T^2} = e^{\frac{\sum_1^i \sum_1^n \ln \frac{x_{ni}+e_{ni}}{p_n} + 0.5 \sum_1^i \sum_1^n \left(\ln \frac{x_{ni}+e_{ni}}{p_n} - R\mu_T\right)^2}{ni}}. \quad (24)$$

The sensitivity function of the expected total demand is

$$Rm_T - m_T = e^{\frac{\sum_1^i \sum_1^n \ln \frac{x_{ni}+e_{ni}}{p_n} + 0.5 \sum_1^i \sum_1^n \left(\ln \frac{x_{ni}+e_{ni}}{p_n} - R\mu_T\right)^2}{ni}} - e^{\frac{\sum_1^i \sum_1^n \ln \frac{x_{ni}}{p_n} + 0.5 \sum_1^i \sum_1^n \left(\ln \frac{x_{ni}}{p_n} - \mu_T\right)^2}{ni}}. \quad (25)$$

Also, the analytical solution of the standard deviation of the total demand with and without error term can be expressed as Rs_T and s_T, respectively:

$$s_T = e^{\mu_T+0.5\sigma_T^2}\sqrt{e^{\sigma_T^2}-1} = e^{\frac{\sum_1^i \sum_1^n \ln \frac{x_{ni}}{p_n} + 0.5 \sum_1^i \sum_1^n \left(\ln \frac{x_{ni}}{p_n} - \mu_T\right)^2}{ni}} \times \sqrt{e^{\frac{\sum_1^i \sum_1^n \left(\ln \frac{x_{ni}}{p_n} - \mu_T\right)^2}{ni}} - 1}, \quad (26)$$

$$R\,s_T = e^{R\mu_T+0.5R\sigma_T^2}\sqrt{e^{R\sigma_T^2}-1} = e^{\frac{\sum_1^i \sum_1^n \ln \frac{x_{ni}+e_{ni}}{p_n} + 0.5 \sum_1^i \sum_1^n \left(\ln \frac{x_{ni}+e_{ni}}{p_n} - R\mu_T\right)^2}{ni}} \times \sqrt{e^{\frac{\sum_1^i \sum_1^n \left(\ln \frac{x_{ni}+e_{ni}}{p_n} - R\mu_T\right)^2}{ni}} - 1}. \quad (27)$$

The sensitivity function of the standard deviation of total demand is:

$$Rs_T - s_T = \left[e^{\frac{\sum_1^i \sum_1^n \ln \frac{x_{ni}+e_{ni}}{p_n} + 0.5 \sum_1^i \sum_1^n \left(\ln \frac{x_{ni}+e_{ni}}{p_n} - \frac{\sum_1^i \sum_1^n \ln \frac{x_{ni}+e_{ni}}{p_n}}{ni}\right)^2}{ni}} \times \sqrt{e^{\frac{\sum_1^i \sum_1^n \left(\ln \frac{x_{ni}+e_{ni}}{p_n} - \frac{\sum_1^i \sum_1^n \ln \frac{x_{ni}+e_{ni}}{p_n}}{ni}\right)^2}{ni}} - 1} \right] - \left[e^{\frac{\sum_1^i \sum_1^n \ln \frac{x_{ni}}{p_n} + 0.5 \sum_1^i \sum_1^n \left(\ln \frac{x_{ni}}{p_n} - \mu_T\right)^2}{ni}} \times \sqrt{e^{\frac{\sum_1^i \sum_1^n \left(\ln \frac{x_{ni}}{p_n} - \mu_T\right)^2}{ni}} - 1} \right]. \quad (28)$$

Note that although Equations (17) and (18) already provide the formula for the estimator and hence also the dependence on the noise terms, the complication in Equations (25) and (28) mainly stem from converting μ_T and σ_T to the mean and variance of the total demand due to that showing the mean and variance could be more intuitive than showing parameters themselves.

The analytical expression of the sensitivity function indicates some characteristics of the estimated results if we design the sensitivity analysis as following:

(1) The specific error: the proportion of the error term e_{ni} to the actual flow x_{ni} on link *n*:

$$e_{ni} = kx_{ni}, \quad n \in N. \quad (29)$$

(2) The systematic error: the number of links that are under a specific error:

$$e_{pi} = kx_{pi}, \quad p \in P,\ P \cup Q = N, \quad (30)$$

$$e_{qi} = 0, \quad q \in Q,\ P \cup Q = N. \quad (31)$$

where P is the set of links with error and Q is the set of links without error. The expected total demand is monotonically increasing with respect to the systematic error and specific error, but the estimated standard deviation of total demand is not monotonic with respect to the systematic error, because from the analytical expression in Equation (28) we can see it is determined by several factors including link proportion, link flow and the specific error. The combined effect of these factors does not satisfy monotonicity. Some discrete scenarios are designed in Table 3 to quantify the impact of error in a simplified way.

3.3 The least squares O–D estimation incorporating the strategic user equilibrium (LSOD)

Using the similar notations in the MLOD method but a different statistical method, the LSOD method minimizes the sum of the squared residual in mean and standard deviation of link flow:

$$\min: \ \mathrm{Obj}\left(m_T, s_T\right) = \sum_{1}^{n}\left(p_n m_T - m_n\right)^2 + \sum_{1}^{n}\left(p_n s_T - s_n\right)^2. \quad (32)$$

Comparing to maximum likelihood method, the least squares method does not require distributional assumptions on link flow. Here the link flow is still assumed to be a random variable, with mean m_T and variance s_T. It is shown that the Hessian matrix is strictly positive, therefore the objective function has unique optimal solution, which can be found when the first derivative of the objective function is equal to zero:

$$\frac{\partial \mathrm{Obj}\left(m_T, s_T\right)}{\partial m_T} = 0 \rightarrow \ m_T = \frac{\sum_1^n p_n m_n}{\sum_1^n p_n^2}\ , \quad (33)$$

$$\frac{\partial \mathrm{Obj}\left(m_T, s_T\right)}{\partial s_T} = 0 \rightarrow \ s_T = \frac{\sum_1^n p_n s_n}{\sum_1^n p_n^2}\ . \quad (34)$$

Similar to the maximum likelihood method, the estimated mean and standard deviation of total demand with error term can be expressed as Rm_T and Rs_T, respectively:

$$R\, m_T = \frac{\sum_1^n p_n \frac{\sum_1^i \left(e_{ni}+x_{ni}\right)}{i}}{\sum_1^n p_n^2}\ , \quad (35)$$

$$R\, s_T = \frac{\sum_1^n p_n R s_n}{i \times \sum_1^n p_n^2}\ . \quad (36)$$

Therefore, the sensitivity expression of estimated results can be obtained:

$$Rm_T - m_T = \frac{\sum_1^n p_n \frac{\sum_1^i \left(e_{ni}+x_{ni}\right)}{i}}{\sum_1^n p_n^2} - \frac{\sum_1^n p_n \frac{\sum_1^i x_{ni}}{i}}{\sum_1^n p_n^2} = \frac{\sum_1^n p_n \sum_1^i e_{ni}}{i \times \sum_1^n p_n^2}\ , \quad (37)$$

$$Rs_T - s_T = \frac{\sum_1^n p_n \left(Rs_n - s_n\right)}{i \times \sum_1^n p_n^2} = \frac{\sum_1^n p_n \left(\sqrt{\frac{\sum_1^i \left[x_{ni}+e_{ni}-\frac{\sum_1^i \left(x_{ni}+e_{ni}\right)}{i}\right]^2}{i}} - \sqrt{\frac{\sum_1^i \left[x_{ni}-\frac{\sum_1^i x_{ni}}{i}\right]^2}{i}}\right)}{i \times \sum_1^n p_n^2}\ . \quad (38)$$

In LSOD method, the sensitivity function of both the mean and standard deviation of total demand illustrates its monotonicity with respect to systematic error and specific error. The sensitivity analysis will prove the monotonicity in both methods in Section 4.

If we compare the optimal solution of both MLOD and LSOD methods, since the logarithm calculus in MLOD method (see Equations (17) and (18)) is only defined for strictly positive numbers, loop counts data needs to be filtered out when either loop counts or link proportions equals zero. Therefore, MLOD may not be used if a network has too many links with zero loop count.

3.4 The bi-level iterative process

Assuming that the StrUE model represents the route choice behavior, we can formulate a bi-level programming problem, where the upper level is either MLOD or LSOD, and the lower level is the StrUE assignment problem. The input to the upper level is the fixed demand proportions, link proportions, and loop detector data, the output is the total demand distribution. The total demand distribution, in conjunction with the fixed demand proportions, will then be used as input to the lower-level problem, which produces the link proportions. The objective functions of both the upper and the lower level are strictly convex, the constraints are also convex, which means the optimization model always have feasible solutions. However, it should be noted that the bi-level programming optimization program may not be convex, because the optimization variable of the upper level is the parameters of the demand distribution whereas that of the lower level is the link proportions, and if the Karush-Kuhn-Tucker (KKT) conditions of lower-level optimization problem were plugged in as the constraints for the upper-level problem, they are no longer convex with respect to the upper-level's optimization variable (the demand distribution parameters). Therefore, the iterative process may not converge. Despite this, when the initial solution is close to the actual one, the solution will very likely be the global optimal because the search domain will be limited to a certain neighborhood of the optimal solution. Experiences have shown that in some cases the results are encouraging (Yang et al., 1992). In this article, a 4-step solution algorithm has been proposed to the bi-level programming:

(1) Initialization: $k = 0$. Start from the prior O–D matrix; obtain the demand proportions q_{rs} and initial values for the mean and the variance of the total demand. Using this total demand distribution as the input and implement the StrUE model to get a set of link proportions $[p]_k$ (which provides the proportions of users choosing each link). Note that q_{rs} will be kept invariant over the bi-level iterations, whereas μ_T^k and σ_T^k will be calibrated.
(2) Optimization: Substituting the link-flow proportion matrix $[p]_k$, solve the upper-level to obtain $\mu_T^{k+1}+$ and σ_T^{k+1} of the total demand.
(3) Simulation: Using μ_T^{k+1} and σ_T^{k+1}, apply the StrUE model to produce a new set of link flow proportions $[p]_{k+1}$.
(4) Convergence test: Calculate the deviation between analytically estimated and observed link flows, and the deviation between analytically estimated total demand distributions of two consecutive iterations, when both of them have met the stopping criterions (the relative change is smaller than a critical value), stop.

4 NUMERICAL RESULTS AND ANALYSIS

4.1 Example of a moderate-scale network: the Sioux Falls network

The objective of the analysis is to test if the MLOD and LSOD can effectively estimate the total demand distribution from day-to-day simulated link flows. The simulation approach consists of artificially determining the total demand distribution (In Sioux Falls network, the expected total demand is provided, the simulated standard deviation of total demand is assumed here.) and generating random link flow samples accordingly. The simulated link flows are used to represent day-to-day observed link flows discussed in the previous parts. The estimated total demand distribution should closely approximate the simulated total demand distribution; the link flow distribution produced by the StrUE model should also closely match the simulated link flows. The analysis reveals both proposed estimation methods will reproduce the desired total demand distribution from the random samples with perturbed prior estimates. The analysis also reflects the scalability of the both MLOD and LSOD to networks of substantial complexity.

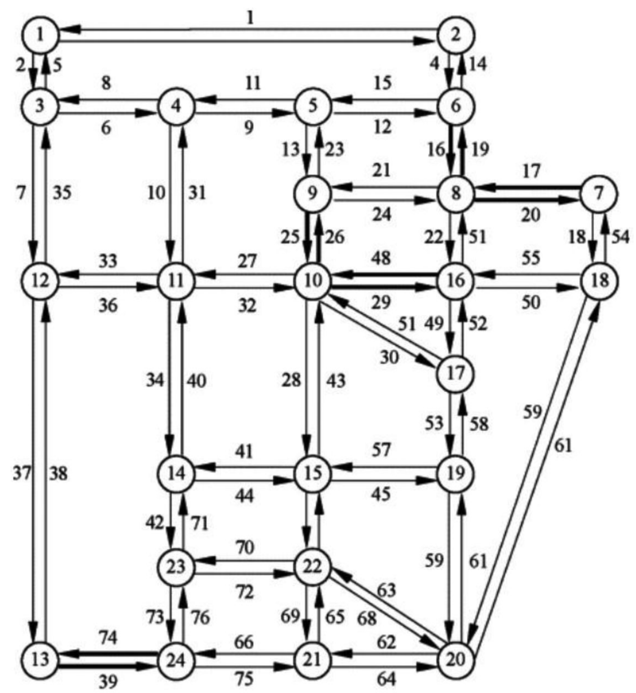

Fig. 1. The Sioux Falls network.

Numerical tests are conducted on the Sioux Falls network (24 nodes and 76 links). The network properties are predefined in Bar-Gera (2012a,b) (see Figure 1). The notations used in this section are defined in Table 1. Each O–D demand is specified as a proportion of the total network demand, therefore the demand for a given O–D pair is the O–D proportion multiplied by the total demand. The BPR function parameters α and β are set to 0.15 and 4.0, respectively.

The simulated link flows are generated by the following way:

(1) The parameters $\mu_T,\ \sigma_T$ are determined for the simulated total demand.
(2) We implement the StrUE based on the simulated total demand distribution and obtain a set of link proportions.
(3) We generate 100 samples of the total demand from the lognormal distribution using $\mu_T,\ \sigma_T$ as parameters and each sampled total demand is assigned to the network using the precalculated link proportions.

Note that by doing this we assume that StrUE can represent users' route choice behavior. In the real world, users' travel behavior is extremely complicated

Table 2
Different scenarios of initial estimation of the O–D matrix

Scenario	*Scenario description*	m_T	s_T
1	$m_T = 0.8m_A$ and cov $= 0.1$	288,480	28,848
2	$m_T = 0.8m_A$ and cov $= 0.3$	288,480	86,544
3	$m_T = 1.2m_A$ and cov $= 0.1$	432,720	43,272
4	$m_T = 1.2m_A$ and cov $= 0.3$	432,720	129,816
5	$m_T = 1.5m_A$ and cov $= 0.1$	540,900	54,090
6	$m_T = 1.5m_A$ and cov $= 0.3$	540,900	162,270
Simulated	$m_A = 360,600$ and cov $= 0.2$		

and is still an open question, which is beyond the scope of this article. The simulated expected total demand of the Sioux Falls network is $m_A = 360,600$, and the coefficient of variation (cov) is equal to 0.2, that is, the standard deviation is 20% of the expected total demand. In Table 2, scenarios 1–6 represent different initial estimates of the total demand distribution. From the simulation all the links are used.

In Figures 2 and 3, the x-axis represents the number of iterations of the bi-level program. Figures 2 and 3 illustrate the estimated mean and standard deviation of total demand in each iteration for the MLOD and LSOD methods, respectively. Each series represents an initial demand scenario. Both figures show that the estimated results converge to the simulated ones within three iterations. This indicates the model's robust performance against biased initial estimates and demonstrates the efficiency in arriving at convergence. The estimated results of the first iteration in both figures are very different from the simulated ones. This is because the link proportions of the first iteration are obtained based on the initial demand scenario specified. The initial estimates in scenarios 1 and 2 are very biased, and as a result, the first iteration results are inaccurate. Both MLOD and LSOD provide a similar estimation of $E(T)$ that is approximately equal to the simulated expected total demand. Note that MLOD always provide an overestimation in $\mathrm{Std}(T)$ as long as initial estimates are biased, but this overestimation is eliminated after the second or third iteration. It is, therefore, necessary to incorporate the bi-level process to reduce the impact of biased initial estimates, especially due to the difficulty in obtaining the standard deviation of demand.

Figures 4 and 5 compare the performance of the estimation methods at the link level for the simulated and analytical results. In Figure 4, the x-axis indicates the simulated expected link flow, whereas the y-axis represents the estimated expected link flow. The estimated link flows are analytically produced by the StrUE model based on the total demand distribution after the convergence criterion has been met. The estimated expected link flows and the corresponding simulated expected link flows are sorted from the smallest to the largest. The R^2 values of both methods are 0.9837 and 0.9917, respectively, which are very close to 1, it indicates that the estimated results closely approximate the simulated expected link flows.

A major strength of the proposed estimation methods, which is an artefact of using the StrUE model for traffic assignment, is the estimation of link flow variation. Since the total demand distribution is calibrated based on day-to-day simulated link flows, it is therefore necessary to compare the estimated standard deviation of link flow to the simulated one. In Figure 5, the estimated standard deviation of link flow is produced by the StrUE model based on the total demand distribution after the bi-level convergence criterion has been met. The x-axis denotes the simulated standard deviation of link flow, whereas the y-axis indicates the estimated one. It is illustrated in Figure 5 that despite the fact that the R^2 value is smaller than that of the expected link flow analysis; the R^2 values of both methods still suggest a satisfying goodness of fit. Note that if the standard deviation of link flow is very high, the estimated results may be more than 20% different from the simulated ones. Such heteroscedasticity is due to that the corresponding link proportions are high. However, both methods can reproduce the link flow distribution if provided an estimated total demand distribution, which indicates its applicability to traffic assignment model.

4.2 Model sensitivity to error in loop detector data for the Sioux Falls network

Loop detector data is prone to error, and the error can be far more complicated in reality, for simplicity, here we design the systematic error and specific error as in Equations (29)–(31) to test the model's sensitivity. The robustness of the proposed estimation methods with respect to both error types is explored using sensitivity analysis. The specific error indicates the significance of the error such as the failure of loop detectors or lack of information on a link; the latter one measures the scale of the error, that is, which links have an error. In this analysis, the specific error is set at 10%, 20%, and 30% respectively, and systematic error is represented as a set of links that have an error, which is shown in Table 3. The impact of different systematic error and specific error is illustrated, by providing the estimated mean and standard deviation of the total demand. Note that as this is a sensitivity analysis, the link proportions derived from the prior demand distribution are assumed to be identical to the simulated ones to eliminate the effect of

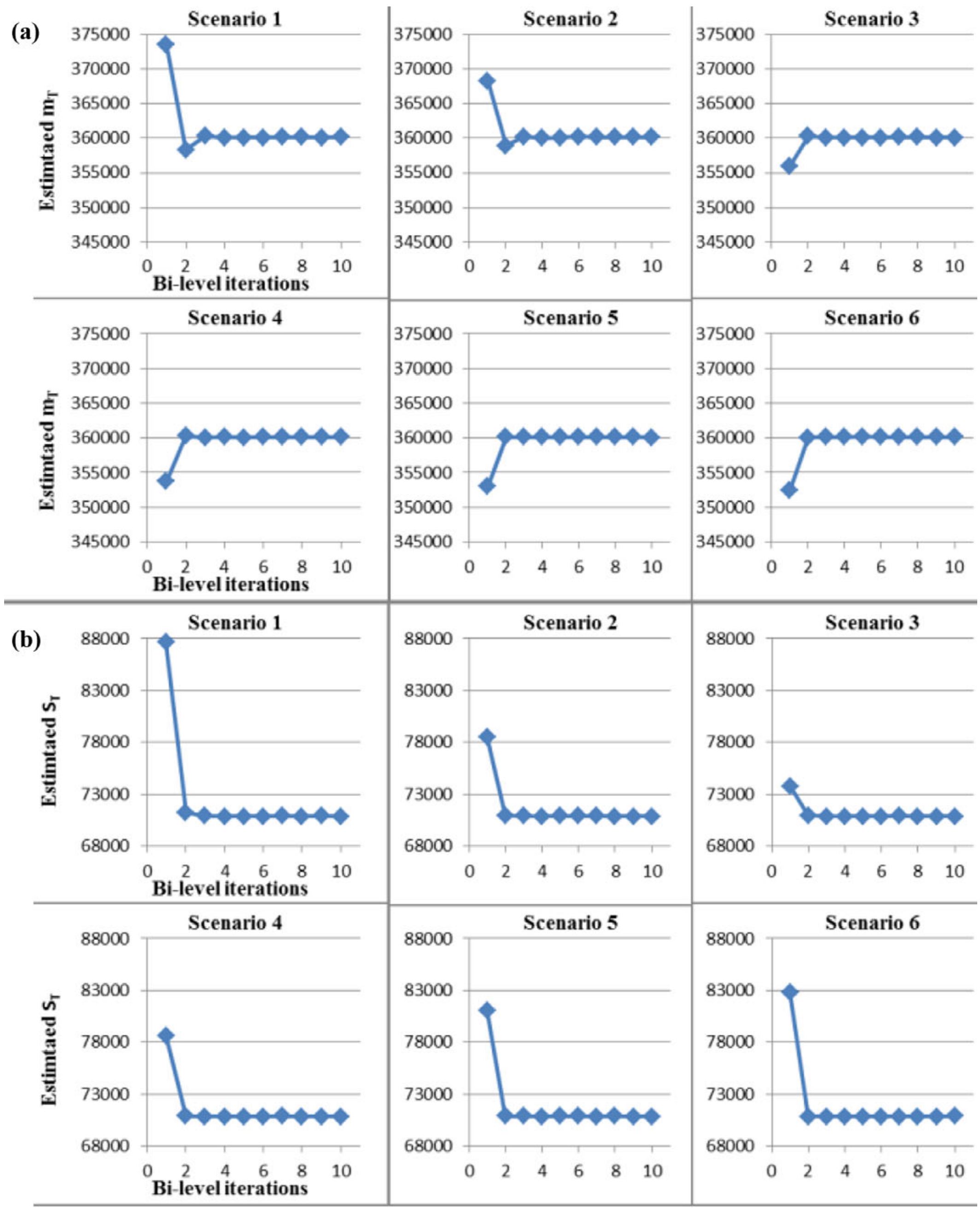

Fig. 2. (a) Estimated expected total demand of MLOD under different scenarios of initial estimation; results of 10 bi-level iterations are presented. (b) Estimated standard deviation of the total demand of MLOD under different scenarios of initial estimation; results of 10 bi-level iterations are presented.

biased prior estimates, that is, link proportions are fixed, since we only focus on the impact of erroneous link flow observations in this part.

In Figure 6, the x-axis represents the systematic error, from 0% (no link has an error) to 100% with an increment of 20%. Each series indicates a different specific error, from 10% to 30% inflation with an increment of 10%. The y-axis starts from the estimated expected total demand without error. It is demonstrated that the estimated expected total demand of both methods rises with the increase in systematic error and specific error; these two error categories have a moderate impact on the estimated expected total demand in both methods. Additionally, LSOD provides estimation closer to the

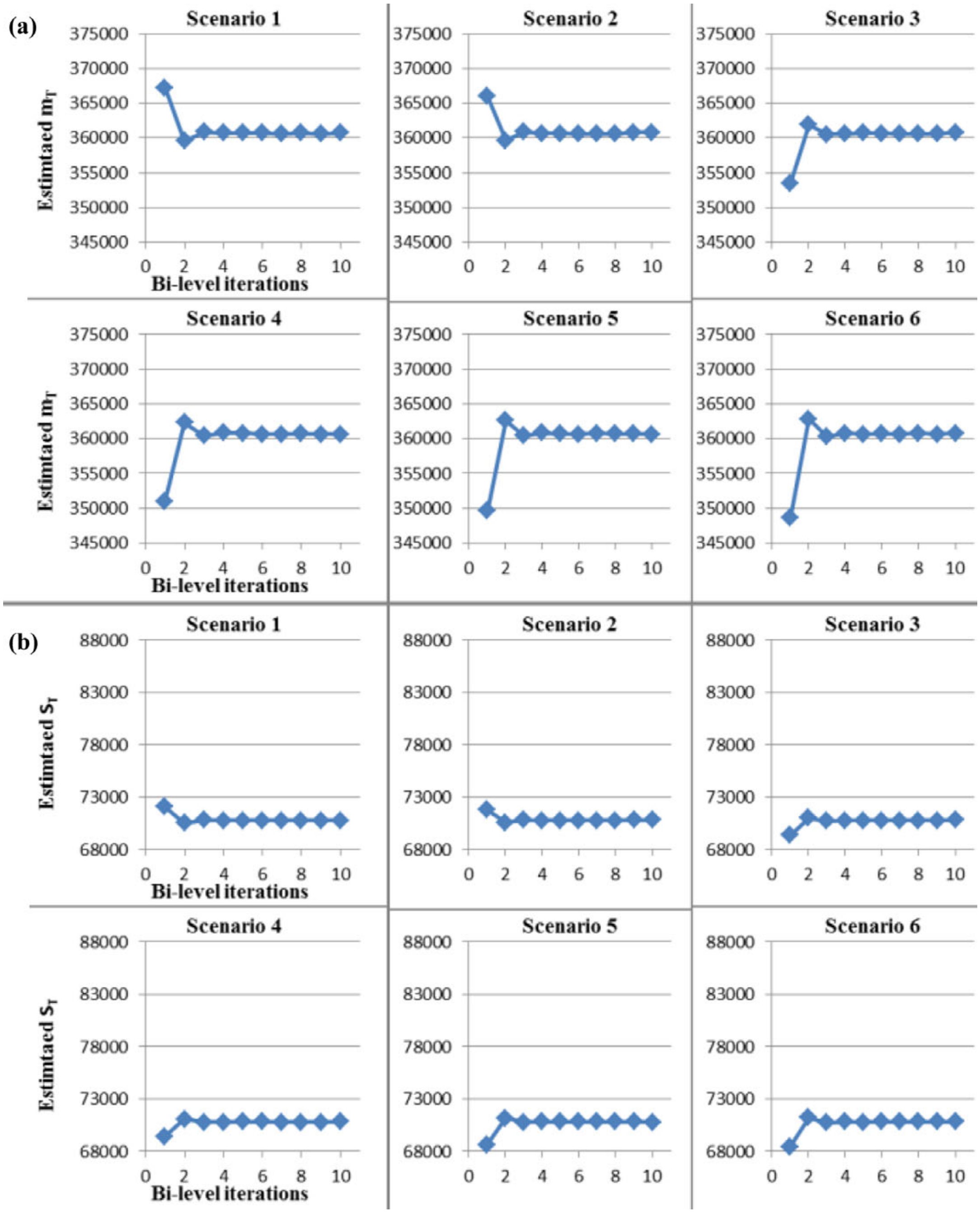

Fig. 3. (a) Estimated expected total demand of LSOD under different scenarios of initial estimation; results of 10 bi-level iterations are presented. (b) Estimated standard deviation of the total demand of LSOD under different scenarios of initial estimation; results of 10 bi-level iterations are presented.

simulated one under low systematic error (20%). Therefore, both systematic error and specific error should be treated equally, and under low systematic error, LSOD can potentially provide a better estimation of estimated expected total demand.

Interestingly, in Figure 7, there is a drop of Std[T] when the systematic error is high in the results of MLOD. This can be explained by the nonlinearity characteristics in the analytical expression of sensitivity. Because from the analytical expression in Equation (28),

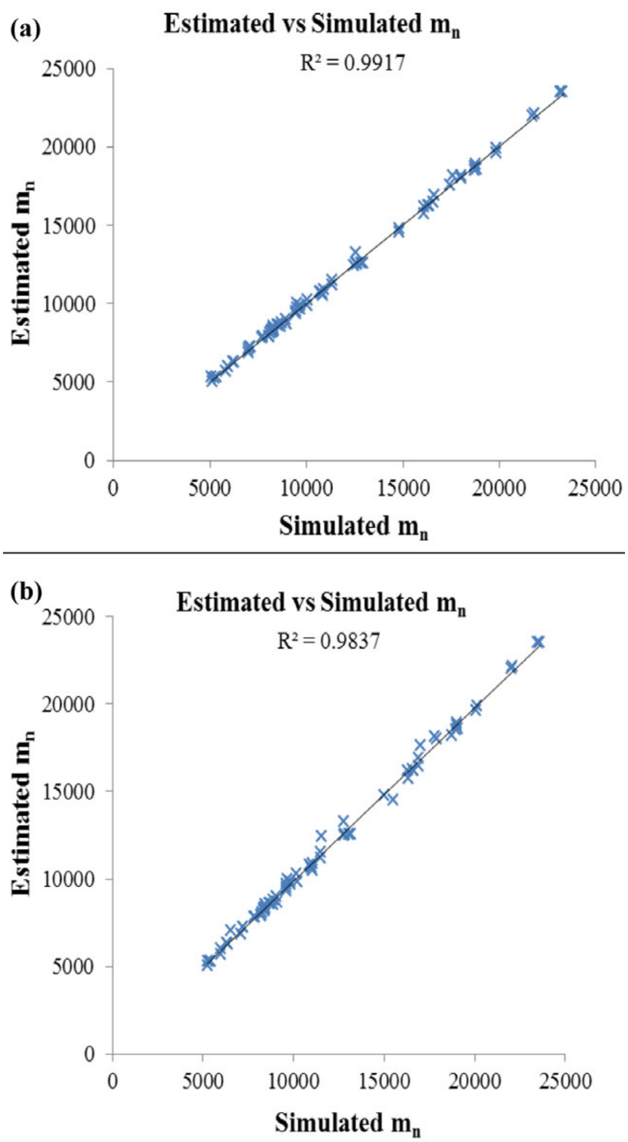

Fig. 4. (a) The estimated and simulated expected link flow comparison of MLOD method, estimated results are produced by the StrUE model based on estimated demand distribution. (b) The estimated and simulated expected link flow comparison of LSOD method, estimated results are produced by the StrUE model based on estimated demand distribution.

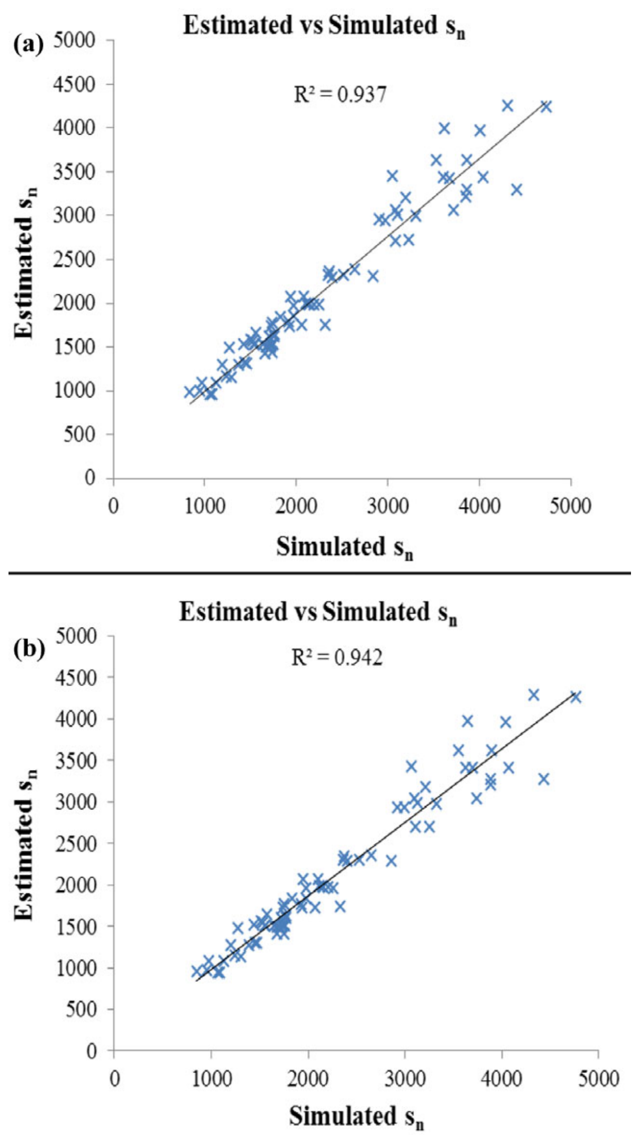

Fig. 5. (a) The estimated and simulated standard deviation of link flow comparison of MLOD method, estimated results are produced by the StrUE model based on estimated demand distribution. (b) The estimated and simulated standard deviation of link flow comparison of LSOD method, estimated results are produced by the StrUE model based on estimated demand distribution.

we can see it is determined by several factors including link proportion, link flow, and the specific error. The combined effect of these factors does not satisfy monotonicity. In LSOD method, Std[T] monotonically increase with systematic error and specific error, and it provides a better estimated Std[T] under all scenarios because of its advantage in the estimation of an overdetermined system.

4.3 Example of a medium-scale network: the Anaheim network

To demonstrate the proposed model's scalability, a numerical analysis is also conducted on the Anaheim network, which consists of 38 zones, 416 nodes, and 914 links. The demand proportions and network properties can be found in Bar-Gera (2012a,b). Units used in the network are: length is in feet; free flow travel time in

Table 3
Different systematic error and specific error

Links with error term (systematic error)	*Total number of samples with error term*	*Specific error*
1-76	7,600	10%, 20%, 30%
16-76	6,100	10%, 20%, 30%
31-76	4,600	10%, 20%, 30%
46-76	3,100	10%, 20%, 30%
61-76	1,600	10%, 20%, 30%

minutes; speed in feet per minute. Parameters for BPR function can be found in Table 4. A similar method as in example one is used to generate the simulated link flows, the actual expected total demand is computed by aggregating the trip table. Figure 8 presents a map of the Anaheim network.

Results of both methods are illustrated in Table 5; same performance measures are depicted as in example

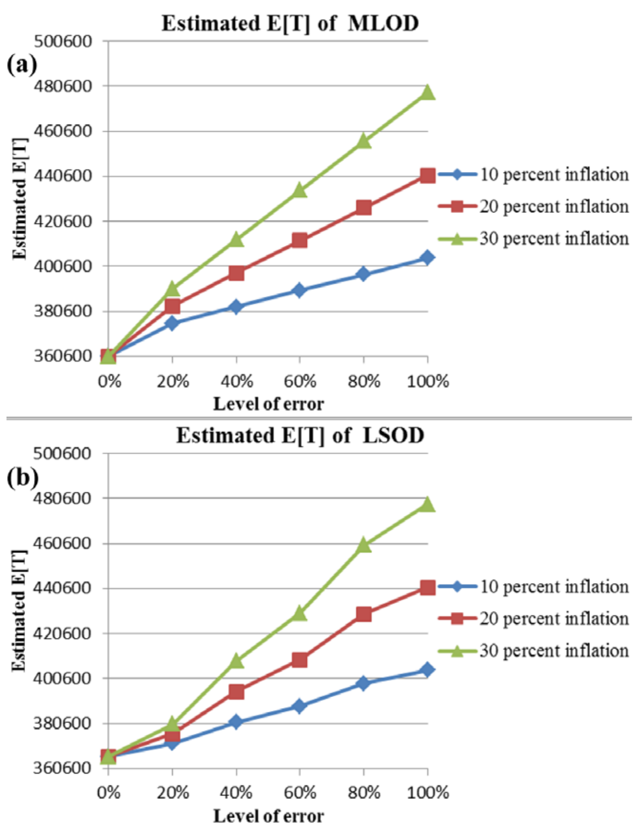

Fig. 6. (a) The estimated expected demand of MLOD method under different systematic error and specific error. (b) The estimated expected demand of LSOD method under different systematic error and specific error.

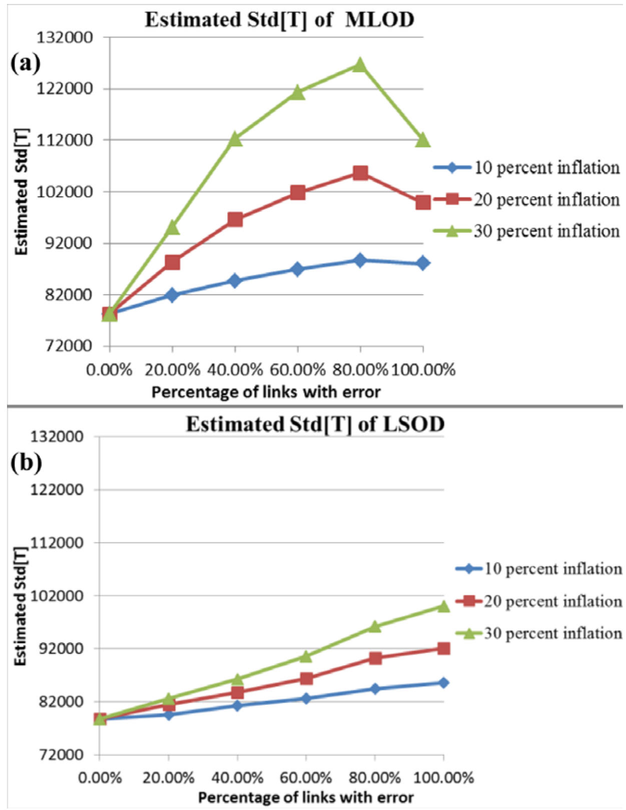

Fig. 7. (a) The estimated standard deviation of the total demand of MLOD method under different systematic error and specific error. (b) The estimated standard deviation of the total demand of LSOD method under different systematic error and specific error.

Table 4
Parameters of BPR function

BPR function
$t_n\,(l_n) = t_{nf}[1 + 0.15 * t_{nf}(\frac{l_n}{C_n})^4]$

one. In addition, to demonstrate the computation efficiency, computation time is recorded (Code is written in MATLAB, the computer used here has the following configurations: Cpu: Intel i7-3770 3.4G Hz quad core, Ram size: 16 GB, Software: Windows 7 enterprise version, note that computation time may vary depending on computer configuration, software version, and other factors). The estimated expected total demand of both methods is statistically indifferent to the actual expected demand. MLOD method slightly underestimates the standard deviation of total demand; although LSOD overestimates the standard deviation of total demand, but the relative error is rather small (relative error is calculated by dividing absolute error by actual

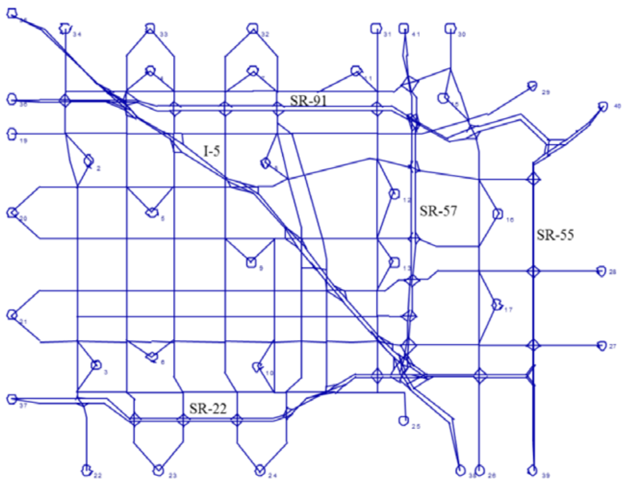

Fig. 8. The Anaheim network.

Table 5
Performance measures of the Anaheim network

Performance measures	*MLOD*	*LSOD*
Estimated m_T	104,730	104,890
Relative error of estimated m_T	0.03%	0.19%
Estimated s_T	20,304	21,433
Relative error of estimated s_T	3.03%	2.36%
R^2 value of estimated and simulated expected link flow	0.991	0.993
R^2 value of estimated and simulated standard deviation of link flow	0.984	0.989
Computation time	179 seconds	156 seconds
Actual demand distribution	$m_A = 104690$ and $s_A = 20939$	

value). R^2 values on the link level demonstrate that the estimated total demand distribution can reproduce a set of link flow close to the simulated ones. Computation time is the time of running the model till convergence and both methods have a short computation time (less than 3 minutes), which proves the model's applicability on larger-scale networks. The computation burden is mainly on the F-W algorithm of the StrUE assignment because each iteration of the F-W algorithm implements the Dijkstra's algorithm to solve the shortest path problem, in the worst-case scenario the computation complexity of Dijkstra's algorithm will be n^2, the total demand calibration process is computationally simple. MLOD takes a moderately longer computation time, due to that some links have no flow on it, and some link proportions may become zero during the bi-level process. In this case, these data need to be filtered out, which increases the computation burden to some extent.

5 CONCLUSION

This article proposes two methods (MLOD and LSOD) to estimate the total traffic demand distribution (trip table) based on day-to-day observed link flows. The model considers link flow variation when estimating the total demand distribution and the StrUE model accounts for the impact of demand volatility in users' route choice decision. A bi-level programming method is included to reduce the impact of biased initial estimates of the total demand distribution; both upper-level and lower-level problems are proved to be strictly convex. A sensitivity analysis is conducted for both MLOD and LSOD methods. A numerical analysis is conducted on the Sioux Falls network and Anaheim network, results for the system level and the link level are similar, and demonstrated the robustness of both MLOD and LSOD methods. In general, both MLOD and LSOD demonstrate scalability with computation efficiency while providing a satisfying estimation of total demand distribution. However, the MLOD method requires nonzero link flow and link proportions, which may limit its applicability in some cases.

Sensitivity analysis shows that the impact of error is predictable. Under the proposed model the problem is overdetermined (unlike the traditional O–D estimation problem), that is, the number of variables to be estimated is far smaller than the number of known constraints, therefore the lack of information on several links will not interfere with the model's implementation. In addition, two consequences can be derived: (1) the advantage of the LSOD method is that it is less sensitive to detector error, which is due to that only the mean and standard deviation of link flows are considered; the impact of errors or outliers will be averaged and thus the estimation tends to be less sensitive to error and (2) despite different objective functions for the two proposed methods, the estimated results without error will be similar because in the proposed models the number of constraints greatly exceeds the parameters to be calibrated, namely the total demand distribution.

Finally, it should be noted that the assumption of fixed O–D demand proportions may limit the model applicability. Therefore, one future research effort will be the generalization of the proposed model, especially on estimating O–D demand independently for each O–D pair. This requires extending the strategic user equilibrium for the case of independently distributed O–D demands; the key contribution will be to account for demand volatility in the assignment model while considering observed link flow variation in O–D demand estimation. This article already provides an insight in light of this. Also, it is valuable to investigate the use of the covariance of loop counts. This can potentially provide much more information than only the link flow distribution. Additionally, dynamic traffic assignment may be integrated into the proposed model. Generally, since the OD estimation problem is a combination of a statistical optimization model and a traffic assignment model, an improvement in either process warrants further research.

REFERENCES

Aerde, M., Rakha, H. & Paramahamsan, H. (2003), Estimation of origin-destination matrices: relationship between practical and theoretical considerations, *Transportation Research Record: Journal of the Transportation Research Board*, **1831**, 122–30.

Airoldi, E. M. & Blocker, A. W. (2013), Estimating latent processes on a network from indirect measurements, *Journal of the American Statistical Association*, **108**(501), 149–64.

Baek, S., Kim, H. & Lim, Y. (2004), Multiple-vehicle origin-destination matrix estimation from traffic counts using genetic algorithm, *Journal of Transportation Engineering*, **130**(3), 339–47.

Bar-Gera, H. (2012a), Transportation Test Problems of Anaheim Network. Available at: http://www.bgu.ac.il/˜bargera/tntp/, accessed December 2015.

Bar-Gera, H. (2012b), Transportation Test Problems of Sioux Falls Network. Available at: http://www.bgu.ac.il/˜bargera/tntp/, accessed December 2015.

Bell, M. G. H. (1983), The estimation of an origin-destination matrix from traffic counts, *Transportation Science*, **17**(2), 198–217.

Bell, M. G. H. (1991), The estimation of origin-destination matrices by constrained generalised least squares, *Transportation Research Part B: Methodological*, **25**(1), 13–22.

Bellei, G., Gentile, G., Meschini, L. & Papola, N. (2006), A demand model with departure time choice for within-day dynamic traffic assignment, *European Journal of Operational Research*, **175**(3), 1557–76.

Bierlaire, M. & Crittin, F. (2004), An efficient algorithm for real-time estimation and prediction of dynamic OD tables, *Operations Research*, **52**(1), 116–27.

Brilon, W., Geistefeldt, J. & Regler, M. (2005), Reliability of freeway traffic flow: a stochastic concept of capacity, in *Proceedings of the 16th International Symposium on Transportation and Traffic Theory*, vol. 125143, College Park, MD.

Cao, J., Davis, D., Vander Wiel, S. & Yu, B. (2000), Time-varying network tomography: router link data, *Journal of the American Statistical Association*, **95**(452), 1063–75.

Cascetta, E. (1984), Estimation of trip matrices from traffic counts and survey data: a generalized least squares estimator, *Transportation Research Part B: Methodological*, **18** (4–5), 289–99.

Castillo, E., Menéndez, J. M., Sánchez-Cambronero, S., Calviño, A. & Sarabia, J. M. (2014), A hierarchical optimization problem: estimating traffic flow using gamma random variables in a Bayesian context, *Computers & Operations Research*, **41**, 240–51.

Chen, A., Chootinan, P. & Recker, W. (2009), Norm approximation method for handling traffic count inconsistencies in path flow estimator, *Transportation Research Part B: Methodological*, **43**(8), 852–72.

Chootinan, P., Chen, A. & Recker, W. (2005), Improved path flow estimator for origin-destination trip tables, *Transportation Research Record: Journal of the Transportation Research Board*, **1923**, 9–17.

Codina, E., Garcıa, R. & Marín, A. (2006), New algorithmic alternatives for the O-D matrix adjustment problem on traffic networks, *European Journal of Operational Research*, **175**(3), 1484–1500.

Cremer, M. & Keller, H. (1987), A new class of dynamic methods for the identification of origin-destination flows, *Transportation Research Part B: Methodological*, **21**(2), 117–32.

Dixit, V., Gardner, L. M. & Waller, S. T. (2013), Strategic user equilibrium assignment under trip variability, *in Proceedings of Transportation Research Board 92nd Annual Meeting*, vol. 9.

Duell, M., Gardner, L., Dixit, V. & Waller, S. (2014), Evaluation of a strategic road pricing scheme accounting for day-to-day and long-term demand uncertainty, *Transportation Research Record: Journal of the Transportation Research Board*, **2467**, 12–20.

Duthie, J. C., Unnikrishnan, A. & Waller, S. T. (2011), Influence of demand uncertainty and correlations on traffic predictions and decisions, *Computer-Aided Civil and Infrastructure Engineering*, **26**(1), 16–29.

Ehlert, A., Bell, M. G. & Grosso, S. (2006), The optimisation of traffic count locations in road networks, *Transportation Research Part B: Methodological*, **40**(6), 460–79.

Fisk, C. S. (1988), On combining maximum entropy trip matrix estimation with user optimal assignment, *Transportation Research Part B: Methodological*, **22**(1), 69–73.

Foulds, L. R., do Nascimento, H. A. D., Calixto, I. C. A. C., Hall, B. R. & Longo, H. (2013), A fuzzy set-based approach to origin–destination matrix estimation in urban traffic networks with imprecise data, *European Journal of Operational Research*, **231**(1), 190–201.

Frederix, R., Viti, F. & Tampère, C. M. J. (2011), Dynamic origin–destination estimation in congested networks: theoretical findings and implications in practice, *Transportmetrica A: Transport Science*, **9**(6), 494–513.

Gan, L., Yang, H. & Wong, S. C. (2005), Traffic counting location and error bound in origin-destination matrix estimation problems, *Journal of Transportation Engineering*, **131**(7), 524–34.

Garcia-Rodenas, R. & Verastegui-Rayo, D. (2008), A column generation algorithm for the estimation of origin–destination matrices in congested traffic networks, *European Journal of Operational Research*, **184**(3), 860–78.

Genschel, U. & Meeker, W. Q. (2010), A comparison of maximum likelihood and median-rank regression for Weibull estimation, *Quality Engineering*, **22**(4), 236–55.

Gentili, M. & Mirchandani, P. (2012), Locating sensors on traffic networks: models, challenges and research opportunities, *Transportation Research Part C: Emerging Technologies*, **24**, 227–55.

Gong, Z. (1998), Estimating the urban OD matrix: a neural network approach, *European Journal of Operational Research*, **106**(1), 108–15.

Hazelton, M. L. (2003), Some comments on origin–destination matrix estimation, *Transportation Research Part A: Policy and Practice*, **37**(10), 811–22.

Hazelton, M. L. (2015), Network tomography for integer-valued traffic, *The Annals of Applied Statistics*, **9**(1), 474–506.

Kamath, K. R. & Pakkala, T. P. M. (2002), A Bayesian approach to a dynamic inventory model under an unknown demand distribution, *Computers & Operations Research*, **29**(4), 403–22.

Kattan, L. & Abdulhai, B. (2006), Noniterative approach to dynamic traffic origin-destination estimation with parallel evolutionary algorithms, *Transportation Research Record: Journal of the Transportation Research Board*, **1964**, 201–10.

Kim, H., Baek, S. & Lim, Y. (2001), Origin-destination matrices estimated with a genetic algorithm from link traffic counts, *Transportation Research Record: Journal of the Transportation Research Board*, **1771**, 156–63.

Kim, J., Kurauchi, F. & Uno, N. (2009), Analysis of variation in demand and performance of urban expressways using dynamic path flow estimation, *Transportmetrica*, **7**(1), 63–84.

Larsson, T., Lundgren, J. T. & Peterson, A. (2010), Allocation of link flow detectors for origin-destination matrix estimation—a comparative study, *Computer-Aided Civil and Infrastructure Engineering*, **25**(2), 116–31.

Lo, H., Zhang, N. & Lam, W. (1999), Decomposition algorithm for statistical estimation of OD matrix with random link choice proportions from traffic counts, *Transportation Research Part B: Methodological*, **33**(5), 369–85.

Lo, H., Zhang, N. & Lam, W. H. (1996), Estimation of an origin-destination matrix with random link choice proportions: a statistical approach, *Transportation Research Part B: Methodological*, **30**(4), 309–24.

Maher, M. (1983), Inferences on trip matrices from observations on link volumes: a Bayesian statistical approach, *Transportation Research Part B: Methodological*, **17**(6), 435–47.

Maus, M., Cotlet, M., Hofkens, J., Gensch, T., De Schryver, F. C., Schaffer, J. & Seidel, C. (2001), An experimental comparison of the maximum likelihood estimation and nonlinear least-squares fluorescence lifetime analysis of single molecules, *Analytical Chemistry*, **73**(9), 2078–86.

Menon, A. K., Cai, C., Wang, W., Wen, T. & Chen, F. (2015), Fine-grained OD estimation with automated zoning and sparsity regularisation, *Transportation Research Part B: Methodological*, **80**, 150–72.

Nie, Y. & Lee, D.-H. (2002), Uncoupled method for equilibrium-based linear path flow estimator for origin-destination trip matrices, *Transportation Research Record: Journal of the Transportation Research Board*, **1783**, 72–79.

Nie, Y., Zhang, H. & Recker, W. (2005), Inferring origin–destination trip matrices with a decoupled GLS path flow estimator, *Transportation Research Part B: Methodological*, **39**(6), 497–518.

Reddy, K. H. & Chakroborty, P. (1998), A fuzzy inference-based assignment algorithm to estimate OD matrix from link volume counts, *Computers, Environment and Urban Systems*, **22**(5), 409–23.

Sherali, H. D., Narayanan, A. & Sivanandan, R. (2003), Estimation of origin–destination trip-tables based on a partial set of traffic link volumes, *Transportation Research Part B: Methodological*, **37**(9), 815–36.

Sherali, H. D., Sivanandan, R. & Hobeika, A. G. (1994), A linear programming approach for synthesizing origin-destination trip tables from link traffic volumes, *Transportation Research Part B: Methodological*, **28**(3), 213–33.

Spiess, H. (1987), A maximum likelihood model for estimating origin-destination matrices, *Transportation Research Part B: Methodological*, **21**(5), 395–412.

Stathopoulos, A. & Tsekeris, T. (2004), Hybrid meta-heuristic algorithm for the simultaneous optimization of the O–D trip matrix estimation, *Computer-Aided Civil and Infrastructure Engineering*, **19**(6), 421–35.

Szeto, W., Jiang, Y. & Sumalee, A. (2011), A cell-based model for multi-class doubly stochastic dynamic traffic assignment, *Computer-Aided Civil and Infrastructure Engineering*, **26**(8), 595–611.

Tebaldi, C. & West, M. (1998), Bayesian inference on network traffic using link count data, *Journal of the American Statistical Association*, **93**(442), 557–73.

Uchida, T. & Iida, Y. (1993), Risk assignment. A new traffic assignment model considering the risk of travel time variation, in *Proceedings of the 12th International Symposium on the Theory of Traffic Flow and Transportation*, Elsevier Science Publishers B.V., Berkeley, CA, United States, pp. 89–105.

U.S. (1964), *Department of Commerce. Bureau of Public Roads. Traffic Assignment Manual Urban Planning Division*, U.S. Department of Commerce, Washington DC.

Vardi, Y. (1996), Network tomography: estimating source-destination traffic intensities from link data, *Journal of the American Statistical Association*, **91**(433), 365–77.

Waller, S. T., Fajardo, D., Duell, M. & Dixit, V. (2013), Linear programming formulation for strategic dynamic traffic assignment, *Networks and Spatial Economics*, 1–17.

Wen, T., Cai, C., Gardner, L., Dixit, V., Waller, S. T. & Chen, F. (2015), A maximum likelihood estimation of trip tables for the strategic user equilibrium model, in *Proceedings of Transportation Research Board 94th Annual Meeting*, No. 15–3826.

Wen, T., Cai, C., Gardner, L., Dixit, V., Waller, S. T. & Chen, F. (2016), A strategic user equilibrium for independently distributed origin-destination demands, in *Proceedings of the Transportation Research Board 95th Annual Meeting*, No. 16–3161.

Wen, T., Gardner, L., Dixit, V., Duell, M. & Waller, S. T. (2014), A strategic user equilibrium model incorporating both demand and capacity uncertainty, in *Proceedings of the Transportation Research Board 93rd Annual Meeting*.

Wong, S., Tong, C., Wong, K., Lam, W. K., Lo, H., Yang, H. & Lo, H. (2005), Estimation of multiclass origin-destination matrices from traffic counts, *Journal of Urban Planning and Development*, **131**(1), 19–29.

Wu, X., Michalopoulos, P. & Liu, H. X. (2010), Stochasticity of freeway operational capacity and chance-constrained ramp metering, *Transportation Research Part C: Emerging Technologies*, **18**(5), 741–56.

Xu, W. & Chan, Y. (1993), Estimating an origin-destination matrix with fuzzy weights: Part II: case studies, *Transportation Planning and Technology*, **17**(2), 145–63.

Yang, H. (1995), Heuristic algorithms for the bilevel origin-destination matrix estimation problem, *Transportation Research Part B: Methodological*, **29**(4), 231–42.
Yang, H., Sasaki, T., Iida, Y. & Asakura, Y. (1992), Estimation of origin-destination matrices from link traffic counts on congested networks, *Transportation Research Part B: Methodological*, **26**(6), 417–34.
Yang, H., Yang, C. & Gan, L. (2006), Models and Algorithms for the screen line-based traffic-counting location problems, *Computers & Operations Research*, **33**(3), 836–58.
Yun, I. & Park, B. B. (2005), Estimation of dynamic origin destination matrix: a genetic algorithm approach, *Intelligent Transportation Systems, Proceedings, 2005 IEEE*, pp. 522–27.
Zhao, Y. & Kockelman, K. M. (2002), The propagation of uncertainty through travel demand models: an exploratory analysis, *The Annals of Regional Science*, **36**(1), 145–63.
Zhou, X. & List, G. F. (2010), An information-theoretic sensor location model for traffic origin-destination demand estimation applications, *Transportation Science*, **44**(2), 254–73.