

A Learning Model for Traffic Assignment: Incorporating Bayesian Inference within the Strategic User Equilibrium Model

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Abstract

This paper addresses adjusted travel route choice in the context of new transport developments and incremental traveller learning. It is assumed that new developments can impact traveller perceptions and adjustments in multiple ways. For instance, if travellers expect a project to significantly increase or decrease overall travel demand they may change their daily route choice based on those new expectations. Further, over time, travellers will learn actual network demand, and adapt their route choice accordingly. In particular, this paper employs a methodological framework to model the day-to-day learning process of road users, and the corresponding system performance over time with a focus on the impact of specific new developments. Travellers assume an initial demand distribution, and incrementally update it based on their day-to-day travel experiences. Bayesian Inference is used to update the travel demand distribution, and the strategic user equilibrium model is used to compute the underlying traffic assignment pattern. Numerical analysis is conducted on a test network to demonstrate the learning process in terms of the perceived travel demand, path choice, and perceived path travel times.

Keywords: Learning, demand uncertainty, strategic user equilibrium, network modelling, Bayesian inference

1. Introduction

New infrastructure development has the potential to fundamentally change the performance of routes throughout a transport network. To assess the impact of new developments, the post-re-equilibration state is commonly employed. However, a critical factor that is often unaccounted for, yet essential to the success of the planning process, is the time taken for users to learn about and adjust to a given change within the system. This study addresses this gap, and proposes a methodological framework which can be used to model the day-to-day learning process of road users, and the corresponding system performance over time. The aim is to help identify an appropriate modelling “horizon”, or time period after a project has been completed, for which the transport system impact can be accurately assessed.

Currently, common practice to determine horizon time periods for future traffic impact assessments are based on the scheduled completion of works and the addition of a fixed time period to account for the users learning and transforming their route choice. The fixed time period of “learning” of users’ is based on standard practice and engineering judgement with no definitive method or approach to its calculation. This study investigates the duration of the learning period of users’ in adjusting to the presence of a new development or infrastructure change within the urban environment. The impact of a new development is inferred as a change to the total travel demand distribution generated between origin and destination pairs throughout the network. The time taken by users to transform their initial perceived travel demand distribution to the actual demand distribution is defined as the learning period. The study uses the Strategic User Equilibrium (StrUE) Model (Dixit et al., 2013) as the foundation of the analysis and incorporates learning through Bayesian inference.

The StrUE model is defined such that "at strategic user equilibrium all used paths have equal and minimal expected cost". For each user present in a given demand scenario, their route choice is based on the distribution of the demand and the route is followed regardless of the realized travel demand on a given day. Therefore, the link flows will not result in an equilibrium state in any particular demand realization, but instead equilibrium exists stochastically across all demand realizations. The StrUE model was proposed to be able to capture the impact of day-to-day demand volatility on reliability, and eventually route choice. In this paper, users will update their perceived distribution curve based on day-to-day travel experience. Studies completed in the recent past make the assumption that the demand fits a log normal distribution (Duell et al., 2014, Wen et al., 2014, Zhao and Kockelman, 2002, Kamath and Pakkala, 2002). The paper is structured as follows. Initially, further details of the background to the problem are presented. The next section provides an explanation of the modelling framework as well as the assumptions made to devise the model. The model is then applied to a sample network and the results of the application are then analysed and discussed. Finally, future extensions and applications of the study are discussed.

2. Backgrounds

Traffic impact assessment and traffic modelling guidelines provide practitioners advice on how to assess the future traffic impacts which will result from the establishment of a new urban development. The guidelines provide detailed methods to forecast the level of travel demand for the future year assessments by either (i) using calibrated and validated regional travel demand models or (ii) by using population and development data (Florida Department of Transport (FDOT), 2014, Roads and Maritime Services (RMS), 2013, The California Department of Transportation (Caltrans), 2002). For example, the Roads and Maritime Services Traffic Modelling Guidelines (2013) states the following; "Planners need to analyse historic data and develop a forecasting methodology appropriate to model future time horizons", providing no clear distinction of how these time horizons are determined. To the authors' best knowledge there is little to any discussion regarding how the actual horizon time period for assessment is determined in practice. To address this gap in the literature this study specifically addresses the impact of new developments on changes to users' route choice over time based on their daily travel experience. The contribution of this study is a methodological framework to determine the horizon time period which should be chosen for project assessment.

Within-day traffic assignment has shown its capability of taking implicitly into account the variability of the flow state along the arc accordingly to any concave fundamental diagram, and modelling real-time traffic (Bellei et al., 2005, Gentile et al., 2005, Helbing et al., 2006). However, most commuters tend to update their commute experiences on a day-to-day basis. Day-to-day travel experiences within a transport network affect future travel decisions, extending from mode choice to route choice along a road transport network (Ben-Elia et al., 2013, Ben-Elia and Shiftan, 2010, Mahmassani and Liu, 1999). Day-to-day dynamics of traffic assignment, which investigates the evolution of travel choices and traffic congestion over time, has been addressed in a number of previous studies (Smith et al., 2014, He et al., 2010, Watling and Hazelton, 2003, Daganzo, 1983, Cascetta and Cantarella, 1991, He and Liu, 2012, Watling and Cantarella, 2013, Wang et al., 2013, Zhao and Orosz, 2014, Hazelton, 2002, Han and Du, 2012, Hazelton and Watling, 2004, Zhang and Nagurney, 1996). Previous research has addressed both deterministic process models (He et al., 2010, Han and Du, 2012, Zhang and Nagurney, 1996) and stochastic process models (Cascetta and Cantarella, 1991, Hazelton, 2002, Hazelton and Watling, 2004). A detailed discussion of the literature within this topic is presented by Watling and Cantarella (2013). A majority of these studies focused on long term traffic equilibration as a result of day-to-day traffic variations and seasonal changes which are expected by the user. Previous studies have also investigated the impact of disruptions (He and Liu, 2012, Wang et al., 2013), providing insight into how long people take to learn about the impacts of a major disruption and how they adjust their routing decisions in the long term. All of these studies provide great insights into

long term user route choice. The work presented in this paper instead focuses on changes in route choice during the adjustment period immediately following the significant change of demand, rather than normal day-to-day conditions.

When considering route choice of road network users, travellers learn about their available routes from their experiences of performing the same trip over an extended period of time. Within the context of this study explicit learning could also potentially arise from the marketing and media of new residential land releases or the opening of new urban infrastructure, this information has the potential to affect how people perceive the travel conditions. There have been a number of approaches to modelling and understanding about learning in a route choice context and how this affects network performance. Bogers et al. (2007) suggests that two types of learning, derived from theories within psychology, play a critical role in day-to-day route choice; implicit or reinforcement based learning and explicit or belief based learning. Implicit learning arises for users as a consequence of travel; a higher relative travel time from a trip would be a negative reinforcement whilst a lower relative travel time would be a positive reinforcement for future decision making (Erev and Barron, 2005). In general, people are habitual decision makers and once an efficient method to complete an activity is devised it is used repeatedly, and this holds true for travel decisions (Jager, 2003). However, when characteristics of the network change, such as the establishment of a new residential development, habitual route choice may not be the most efficient method resulting in implicit learning. Additionally, in a transport context, explicit learning will also occur when users gain knowledge from information sources and their beliefs of these information sources (Arentze and Timmermans, 2003).

Controlled laboratory experiments using repeated route choice games have been conducted to understand users' learning behaviour and results have been adapted to discrete choice models (Ben-Elia and Shiftan, 2010, Cominetti et al., 2010, Bogers et al., 2005). In particular, Ben-Elia and Shiftan (2010) presented that initial risk seeking behaviour in route choice transforms into risk averse behaviour as learning progresses which is consistent with the findings of Arentze and Timmermans (2005). Experimental approaches provide the ability to investigate dynamic system evolution and the behavioural implications of users' day-to-day choices. However, a shortcoming with this method is that there is difficulty in resolving the biases that may occur within the simulated environment as compared to the real environment (Chen and Mahmassani, 2004). In terms of econometric modelling, Horowitz (1984) developed an updating version of EUT to analyse repeated travel choice situations using a weighted average approach in calculating the perceived travel cost of a route. Further studies have also used this concept where the route choice is determined by a process of adaptive learning where the information affects the utility of the route and the knowledge of the road network for future decisions (Mahmassani and Liu, 1999, Srinivasan and Mahmassani, 2003, Mahmassani et al., 1986). De Palma and Marchal (2002) investigated day-to-day learning using an exponential Markov process representing learning; however this model was not validated with empirical data. A drawback of all the perception updating methodologies described above is that they do not capture drivers' uncertainty in their estimation of travel time which can be accounted for using a Bayesian updating approach (Jha et al., 1998, Chen and Mahmassani, 2004). Jha et al. (1998) uses a Bayesian updating model to capture the mechanism by which travellers update their day-to-day travel time perceptions based on previous experiences and information from ATIS systems. Chen and Mahmassani (2004) extend the use of Bayesian Inference by also considering heuristics to trigger and terminate the learning process to depict a users' salience to new information. The social impact under uncertainty on traffic was also demonstrated (Sunitiyoso et al., 2011), the parameterization of modelling the learning, or evolution of urban network was discussed as an extension of physical rules (Helbing and Nagel, 2004). All these studies provided a background in developing the methodology of this study. Specifically, this study utilizes Bayesian Inference to model the learning process within the StrUE model, which explicitly incorporates uncertainty into the traffic assignment problem. The analysis of travel behaviour

in uncertain conditions has historically focussed on three economic theories, Expected Utility Theory (EUT), Prospect Theory (PT) and Regret Theory (Ben-Elia et al., 2013). Research and models developed using these theoretical frameworks provide great insight into one-shot decision making where the outcome of one decision has no relationship to the next (Arentze and Timmermans, 2005). In contrast, the focus of this study is to incorporate experiential information into the users' decision process.

Travel demand is a main factor that affects travel time on a network. Though demand data is difficult to obtain, the expected demand and an estimate for the distribution of travel demand can be obtained through loop detector data, household survey data and through many other approaches. However, there is a certain degree of uncertainty that exists with these estimations. This study incorporates this uncertainty by providing partial information regarding the demand distribution to the user as well as including a perception component which interprets the user's confidence level of their estimation of the travel demand. A Bayesian Inference Model was implemented to update users' perceived travel demand distributions based on previous travel experiences, which contrasts previous studies which investigated the update of travel time perceptions. Furthermore, the study utilizes the strategic user equilibrium (StrUE) model (Dixit et al., 2013) to determine the traffic assignment pattern corresponding to each step in the learning process, and quantify various system performance metrics. The static version of the StrUE model, which has been developed in a static and dynamic context (Waller et al., 2013), was specifically selected because it offers a way to incorporate day-to-day demand variability. The importance of accounting for demand volatility has also been discussed in many previous papers (Clark and Watling, 2005, Duthie et al., 2011, Uchida and Iida, 1993); the model ensures that users recognize the uncertainty or variability in travel time to their destination and rationally choose routes while considering all possible demand scenarios from a known (or perceived) distribution. In addition, the model provides the link flow variability as a result of the demand volatility.

3. Problem Formulation

In this section we describe the Bayesian inference process which is used to model the learning behaviour of users. The underlying traffic assignment model implemented in this model framework is the strategic user equilibrium (StrUE) assignment model which is described in detail in (Dixit et al., 2013). A brief description of this assignment model is also included for completeness in this section. Table 1 lists the notation used in this section.

TABLE 1 Summary of notation

N	Node (index) set
A	Link (index) set
K_{RS}	Path set
f_n	Proportion of total demand on link n ; $f = (\dots, f_n)$
t_n	Travel time on link n ; $t = (\dots, t_n, \dots)$
t_{nf}	Free flow travel time on link n
C_n	The capacity on link n
p_k^{rs}	Proportion of flow on path k , connecting OD pair r - s , must be non-negative; $p^{rs} = (\dots, p_k^{rs}, \dots)$; $p = (\dots, p^{rs}, \dots)$
c_k^{rs}	Travel time on path k connecting O-D pair r - s ; $c^{rs} = (\dots, c_k^{rs})$; $c = (\dots, c^{rs}, \dots)$
q_{rs}	Fraction of total trip demand that is between OD pair r - s ; $1 = \sum_{r,s} q_{rs}$
T_a	Actual total trip demand
T_p	Users' perceived total trip demand
$g(T)$	The probability distribution function of the total trips, T
x_i	The realized total trip demand observed by users on day i .
$P(x y)$	The conditional probability of x given y .

$\delta_{a,k}^{rs}$	Indicator variable	$\delta_{a,k}^{rs} = \begin{cases} 1 & \text{if } a \text{ is included in path } k \\ 0 & \text{otherwise} \end{cases}$	$(\Delta^{rs})_{a,k} = \delta_{a,k}^{rs}$
Δ	$\Delta = (\dots, \Delta^{rs}, \dots)$		
$E()$	Expected value function		
$Var()$	Variance function		
$Std()$	Standard deviation function		

The StrUE model which is employed in this paper is defined such that "at Strategic User Equilibrium all used paths have equal and minimal *expected* cost". The model explicitly accounts for day-to-day demand uncertainty and assumes that users make strategic routing choices considering full knowledge of the demand distribution. For a given OD pair the StrUE model provides a set of fixed path proportions, p_k^{rs} . For each user present in a given demand scenario, the chosen route is then followed regardless of the realized travel demand on a given day. The strategic assignment model therefore produces link flows which will not result in a state of network equilibrium under any particular demand realization. The mathematical formulation for the StrUE model is presented below:

$$\text{Minimize } z(f) = \sum_{n \in A} \int_0^{f_n} \int_0^\infty t_n(f_n T) g(T) dT df \quad [1]$$

Subject to:

$$\sum_k p_k^{rs} = q_{rs} \quad \forall r, s \quad [2]$$

$$p_k^{rs} \geq 0 \quad \forall r, s \quad [3]$$

$$f_n = \sum_r \sum_s \sum_k p_k^{rs} \delta_{a,k}^{rs} \quad \forall r, s \quad [4]$$

The link travel time function, $t_n(f_n T)$, is modelled after the Bureau of Public Roads cost function (U.S, 1964), where α and β are the parameters for the BPR function:

$$t_n(flow) = t_{nf} \left[1 + \alpha t_{nf} \left(\frac{f_n T}{c_n} \right)^\beta \right] \quad [5]$$

The pdf for the total trips, $g(T)$, is assumed to follow a lognormal distribution. The positiveness of the log normal distribution and the ease of determining conjugate priors necessary for the Bayesian inference process ease the computational process and as such are assumed for this study as well. However, it must be noted that other distributions could be assumed, however significant mathematical manipulation would be required to ensure positivity and apply Bayesian inference. With StrUE, the expected link travel time and variability of link travel time can be shown to be strictly a manifestation of travel demand uncertainty, and analytically defined as follows:

$$E(t_n) = t_{nf} + \alpha t_{nf} \left(\frac{f_n}{c_n} \right)^\beta M_\beta \quad [6]$$

$$var(t_n) = E(t_n^2) - [E(t_n)]^2 = \alpha^2 t_{nf}^2 \left(\frac{f_n}{c_n} \right)^{2\beta} [M_{2\beta} - M_\beta^2] \quad [7]$$

Equation 6 defines the expected link travel time and equation 7 defines the variance of link travel time, where the M_β is the β_{th} moment of the demand distribution and can be found analytically using the moment generating function. For a given demand distribution, the StrUE assignment problem can be solved using any algorithm capable of solving the static traffic assignment problem. In this work the Frank-Wolfe algorithm is implemented to compute the link proportions. The difference is we use equation 6 with the moment generating function to calculate and update the expected travel time instead of a constant flow, hence the shortest path cost will change accordingly.

As stated previously, the main contribution of this work is that the learning process is incorporated into the novel strategic user equilibrium model. To incorporate learning, the user's *perceived* demand distribution is used to compute the link costs in StrUE, thus resulting in a given system assignment pattern. Furthermore, the perceived demand distribution is assumed to change over time based on knowledge gained by users through their past travel experiences. Every time the perceived demand distribution is updated, the link costs functions will change, resulting in a new set of equilibrium-based path choices. To update the perceived distribution a Bayesian inference process was implemented, which is described below. The learning model with underlying StrUE assignment is hereby referred to as L_StrUE.

Firstly, two demand distributions were defined, i) the actual demand distribution and ii) the perceived demand distribution. The actual distribution represents the true state, from which day-to-day demands are sampled. The perceived distribution is what the users assume to be true at the time. It was assumed that the actual distribution does not change during the timeframe of concern. The perceived distribution is assumed to initially underestimate or overestimate the expected trip demand and variance. Both distributions were assumed to follow lognormal distributions with known, but different, parameters. The actual demand distribution is defined as $T_a \sim \text{Lognormal}(\mu_a, \rho_a)$, and the perceived demand distribution is defined as $T_p \sim \text{Lognormal}(\mu_p, \rho_p)$. Note that μ and ρ are simply parameters of the lognormal distribution, and have a direct relation to the mean and variance of the lognormal distribution, defined in equation 8 and 9, respectively. These equations represent the mean and variance of the total trip demand distribution.

$$E(T) = e^{\mu + \frac{1}{2\rho}} \quad [8]$$

$$\text{var}(T) = \left(e^{\frac{1}{\rho}} - 1\right) [E(T)]^2 \quad [9]$$

Because providing the mean and variance of total trip demand is more intuitive than simply assuming the corresponding lognormal parameters, the lognormal parameters μ_a and ρ_a were back calculated based on the actual demand distribution, $E(T_a)$ and variance of $\text{Var}(T_a)$. It was further assumed i) that the perceived location parameter μ_p is identical to the location parameter of the actual demand distribution, μ_a , ii) it is known by the users, and iii) remains fixed over the course of the learning process. This is based on the assumption that users have some level of prior knowledge which they base their initial perceived distribution on (i.e. they are not unfamiliar drivers). The assumption also allows us to compute the perceived expected demand, $E(T_p)$ and variance of perceived demand $\text{Var}(T_p)$, for any precision parameter, ρ_p . Note that the assumption of identical location parameters, and the assumption below of the gamma distribution are made because they can reduce the computation complexity without compromise in the investigation of the learning process,, otherwise numerical integration may have to be used in every iteration.

The learning process is modelled using Bayesian Inference to update the precision parameter, ρ_p , based on users' previous travel experiences. The initial perceived precision parameter ρ_p is assumed to be a random variable, and follows a gamma distribution, $\rho_p \sim \text{Gamma}(\alpha, \beta)$. The Gamma distribution is capable of describing various kinds of probability curves and is always positive. From Bayesian inference, the posterior distribution is a function of both the prior distribution and the likelihood function. The posterior distribution of the precision, ρ_p , given that a set of data t are observed, is:

$$P(\rho_p | t) = \frac{P(t | \rho_p)P(\rho_p)}{\int P(t | \rho_p)P(\rho_p)d\rho_p} \quad [10]$$

The gamma distribution is the conjugate prior of the lognormal likelihood function, i.e. if the actual demand has a lognormal distribution, from Bayesian inference, the closed form probability distribution function of the posterior distribution exists, and is also a gamma distribution. The precision variables for the prior and posterior distributions are thus defined as $\rho_{prior} \sim \text{Gamma}(\alpha_0, \beta_0)$ and $\rho_{posterior} \sim \text{Gamma}(\alpha, \beta)$, respectively. The α and β parameters are initialized as (α_0, β_0) , and updated each day based on users' travel experience, as defined below:

$$\alpha = \alpha_0 + \frac{n}{2}, \quad [11]$$

$$\beta = \beta_0 + \frac{\sum_{i=1}^n (\ln x_i - \mu_p)^2}{2} \quad [12]$$

The expected precision and variance of precision can therefore be defined in terms of α and β as follows:

$$E(\rho_{posterior}) = \frac{\alpha}{\beta} \quad [13]$$

$$\text{Var}(\rho_{posterior}) = \frac{\alpha}{\beta^2} \quad [14]$$

The variance of the precision can be interpreted as the confidence level of a user group, and reflects their willingness to adapt their route choice (i.e. update the perceived demand distribution) based on past travel experiences. A low precision variance represents a user group whom is more confident in their initial perception of the travel conditions, and is therefore going to be less willing to change their perception based on travel experiences. A higher precision variance represents a user group whom is less confident in their initial perception of the travel conditions, and is therefore going to be more willing to adapt their route choice based on past travel experiences. The impact of this variable is illustrated in a sensitivity study presented in the numerical analysis section.

In the analysis conducted, a single iteration is equivalent to a day during which users commute to work. Each day the users will select a route based on their perceived demand at the time. On the same day, a demand will be realized, which is sampled from the actual demand distribution, resulting in a set of path flows on the network and the consequent link and path travel times. The users observe these travel times, and update their perceived demand curves accordingly, by updating parameters, α and β . The updated parameters are then used to compute the updated perceived precision parameter, ρ_p , using the expected posterior precision from equation 13. At the end of each iteration the updated perceived demand distribution will be:

$$T_p \sim \text{Lognormal}(\mu_p, E(\rho_{posterior})). \quad [15]$$

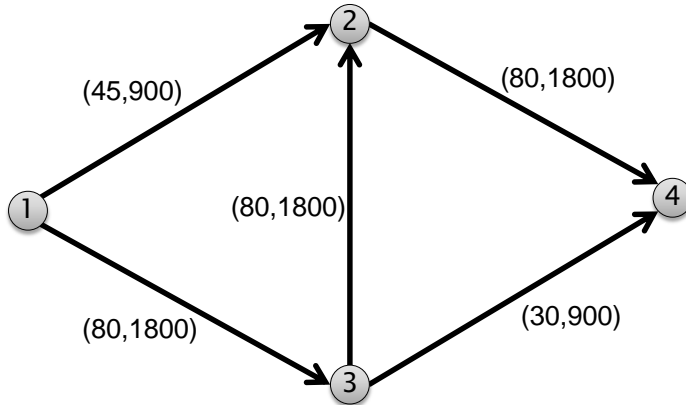
At the end of the entire learning process the users' final perceived demand distribution will be defined by $T_p \sim \text{Lognormal}(\mu_p, \rho_{posterior})$, and is expected to have converged from the initial perceived distribution, $T_p \sim \text{Lognormal}(\mu_p, \rho_{prior})$ to the actual distribution, $T_a \sim \text{Lognormal}(\mu_a, \rho_a)$. Note that because α_0 and β_0 determine the prior precision variable, assuming the initial perceived distribution is equivalent to assuming a mean and variance of the precision variable. To further explore the impact of the initial perception distribution on the learning process, the next section illustrates the convergence behaviour of L_StrUE for a range of initial perceived distributions modelled on a test network.

4. Numerical Analysis

The developed L_StrUE model has been demonstrated using on a test network with 4 nodes and 5 links (Braess Network) as presented in Figure 1. The assumed network properties are

defined within Figure 1, free flow travel times and capacity for each link are shown in parentheses in units of miles per hour and vehicles per hour, respectively. In addition, the BPR parameters α and β for all links are equal to 0.15 and 4, and the length of each link is 1 mile.

FIGURE 1: Test network



The analysis investigated the sensitivity of the network performance regarding two main components: *i*) different initial perceived demand distributions (i.e. how accurate drivers' initial perception is relative to the actual travel demand distribution) and *ii*) the impact of an increasing precision variance (i.e. confidence level of the drivers in regards to their initial perception). The purpose of conducting these two sensitivity analyses was firstly, to compare the learning process when the initial perceived and actual demand distribution varied, and secondly, reveal the role of the precision variance in the learning process. For all scenarios evaluated the actual demand curve was fixed, with a mean demand of 2700 and standard deviation of 270, or 10% of the mean. Twelve scenarios were selected in a systematic fashion representing different combinations of perceived overestimation believed by users, and precision variance (confidence levels). The scenario selection was based on the idea that road network users' would have knowledge of a new (recent) development and as a result perceive conditions which are inflated relative to the historical traffic conditions. The precision variance levels were chosen to demonstrate the system impact of users' willingness to adapt. The set of scenarios assessed in this analysis are presented in Table 2. The perceived standard deviation of total trips, $STD[T_p]$, is presented as a percentage of the perceived expected total trips $E[T_p]$. For each scenario 2000 iterations were run in order to capture the entire learning process.

TABLE 2: Scenarios assessed for numerical analysis

Scenario	Variance of precision	$E[T_p]$	$STD[T_p]$
1	0.1	2835	34%
2	0.1	3240	67%
3	0.1	3510	84%
4	0.1	4050	113%
5	0.2	2835	34%
6	0.2	3240	67%
7	0.2	3510	84%
8	0.2	4050	113%
9	0.3	2835	34%
10	0.3	3240	67%
11	0.3	3510	84%
12	0.3	4050	113%

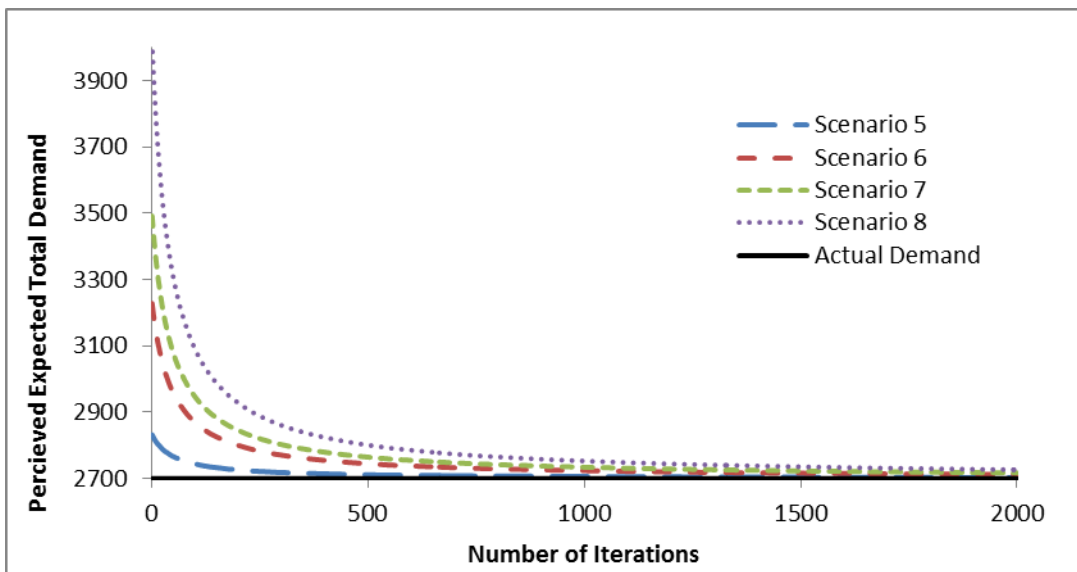
4.1 System Level Performance

A system level performance assessment was conducted to obtain an understanding of the convergence of the L_StrUE model under the different scenarios tested. The purpose of the analysis was to identify system level performance metrics for different initial perceived demand distributions and ii) variance of the precision variable. In the analysis presented the time to convergence provides a proxy for the time taken for users' to learn the actual travel demand conditions, which is one of the main objectives of this study. It is however important to note, the numerical results of the L_StrUE model are specific to this case study, and at this point cannot be extrapolated to alternative network structures. The main contribution of this work is the proposed framework for modelling the learning process travellers go through due to changes to the network conditions. This study also serves to demonstrate various potential applications of the model. Throughout the following sections, results from a subset of the scenarios are presented, which are representative of the trends observed across all the scenarios tested.

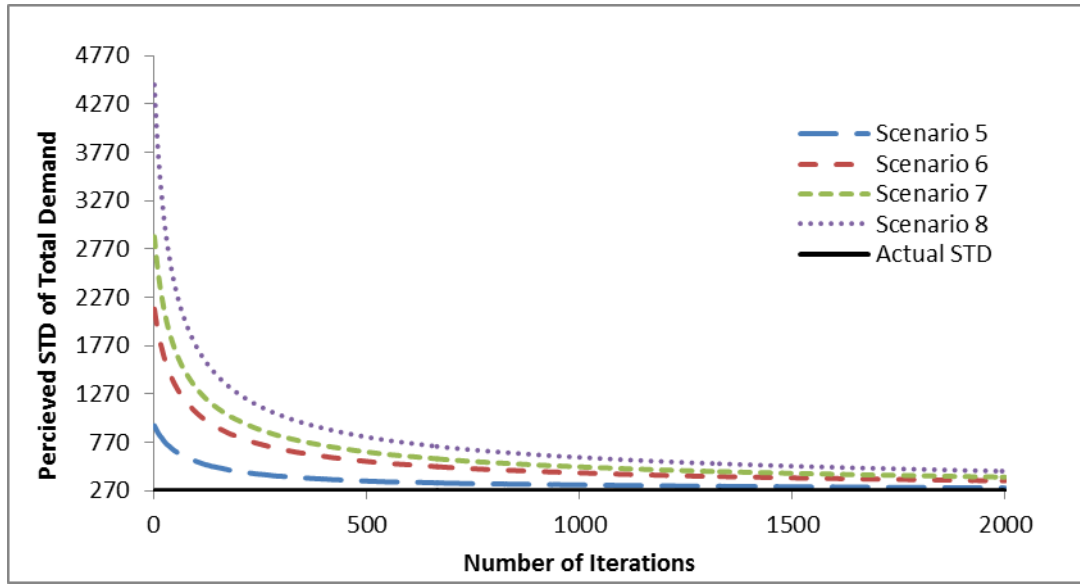
4.1.1 Sensitivity to Initial Perceived Demand Distribution

The convergence of the perceived expected demand over the learning period is illustrated in Figure 2(a). The horizontal axis represents the learning time, or the number of learning iterations, while vertical axis represents users' perceived expected total demand. The figure shows Scenarios 5 through to 8 which consider the range of initial perceived demand curves presented in Table 1, and a fixed precision variance of 0.2. The horizontal line in the figure depicts the actual total demand. The figure illustrates the convergence of all the scenarios; after 365 iterations (which can be interpreted as a year of daily travel) the perceived demand is within 5% of the actual demand. The initial perceived demand distribution is shown to have significant impact on the learning process. Figure 2.a reveals the most inflated initial perceived distribution has taken almost twice as long to converge than the least inflated scenario. Figure 2(b) presents the convergence of the perceived standard deviation of demand over the learning period, illustrating a similar trend to what was observed in Figure 2(a). Both these figures suggest that when people's perceived demand distributions more closely reflect the actual demand distribution, the learning time is reduced significantly. This can potentially be a source of information provided to users in order to reduce the level of learning required within a system.

FIGURE 2: Illustration of Convergence of a) Perceived Expected Demand b) Perceived Variation of Demand under different initial perceived demand distributions.



(a)



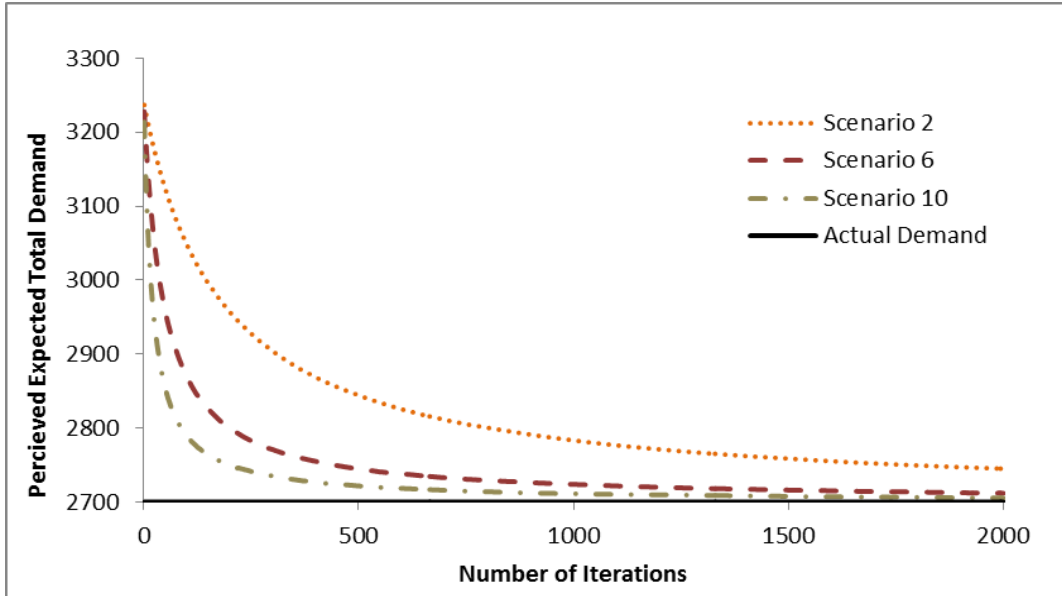
(b)

4.1.1 Sensitivity to Precision Variance

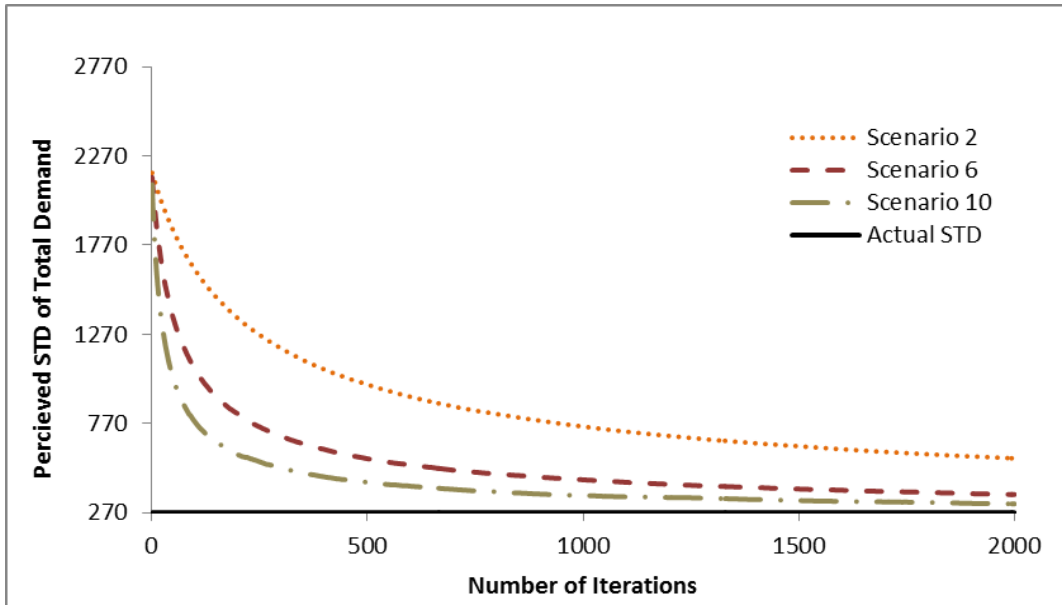
The impact of increased precision variance on the convergence of the L_StrUE model is presented in this section. The convergence of perceived expected demand and perceived standard deviation of demand are presented in Figure 3(a) and Figure 3(b), respectively. The figures illustrate Scenarios 2, 6 and 10 corresponding to a precision variance of 0.1, 0.2 and 0.3, where the initial perceived demand distribution remains fixed.

Figure 3(a) and 3(b) clearly indicate convergence of the perceived expected value and standard deviation of the total demand to the actual distribution values. However the rate of convergence is significantly affected by the value of the precision variance. The lowest precision variance results in a considerably slower convergence when compared with the other two scenarios. Again, the lower precision variance represents a user group that is more confident in their initial perception, and therefore less willing to change their route choice. Similarly, a higher precision variance indicates a user group who is less certain about the prevailing traffic conditions and therefore less confident in his/her initial perception of the travel demand. These users can possibly be categorised as “new road users” or an “unfamiliar road users”, as they are more willing to update their route choice based on previous travel experiences, and therefore learn the actual demand faster, as illustrated by increased rate of convergence of the L_StrUE model. An alternative explanation of this behaviour is that the user is a “fast learner” and someone who is aware of the presence of the new development and has rationalised the potential effect on traffic and is willing to adjust his travel patterns. In contrast, users with a lower precision variance could be classified as “stubborn users” who are determined that their initial perceptions reflect the actual traffic conditions, and refuse to accept the changed resulting from a new development. These users exhibit a slower rate of learning, as illustrated by decreased rate of convergence of the L_StrUE model. The results from the L_StrUE model therefore provide a behavioural intuition regarding the precision variance.

FIGURE 3: Illustration of Convergence of a) Perceived Expected Demand and b) Perceived Variation of Demand under different precision variances.



(a)



(b)

4.2 Path Level Analysis

In addition to evaluating the learning process at the system level, a path level assessment was conducted to explore the impact of the learning process on user route choice under the different scenarios tested. Of specific interest was the changes to path flows and path travel times over time as users learned the actual conditions of the network. Similar sensitivity analysis was conducted to explore the changes in path flows over time relative to *i)* the variation of the initial perception distribution and *ii)* variance of the precision variable.

As described in the methodology section, the path assignment is computed using the StrUE model. It is important to note that the StrUE model provides unique link proportions and ultimately unique link flows, not unique path flows. However, for the test network used within this study, it was possible to obtain path performance statistics because there were distinct links associated to individual paths. Path-based statistics are presented instead of link level statistics because they provide a more intuitive illustration of the network performance. As

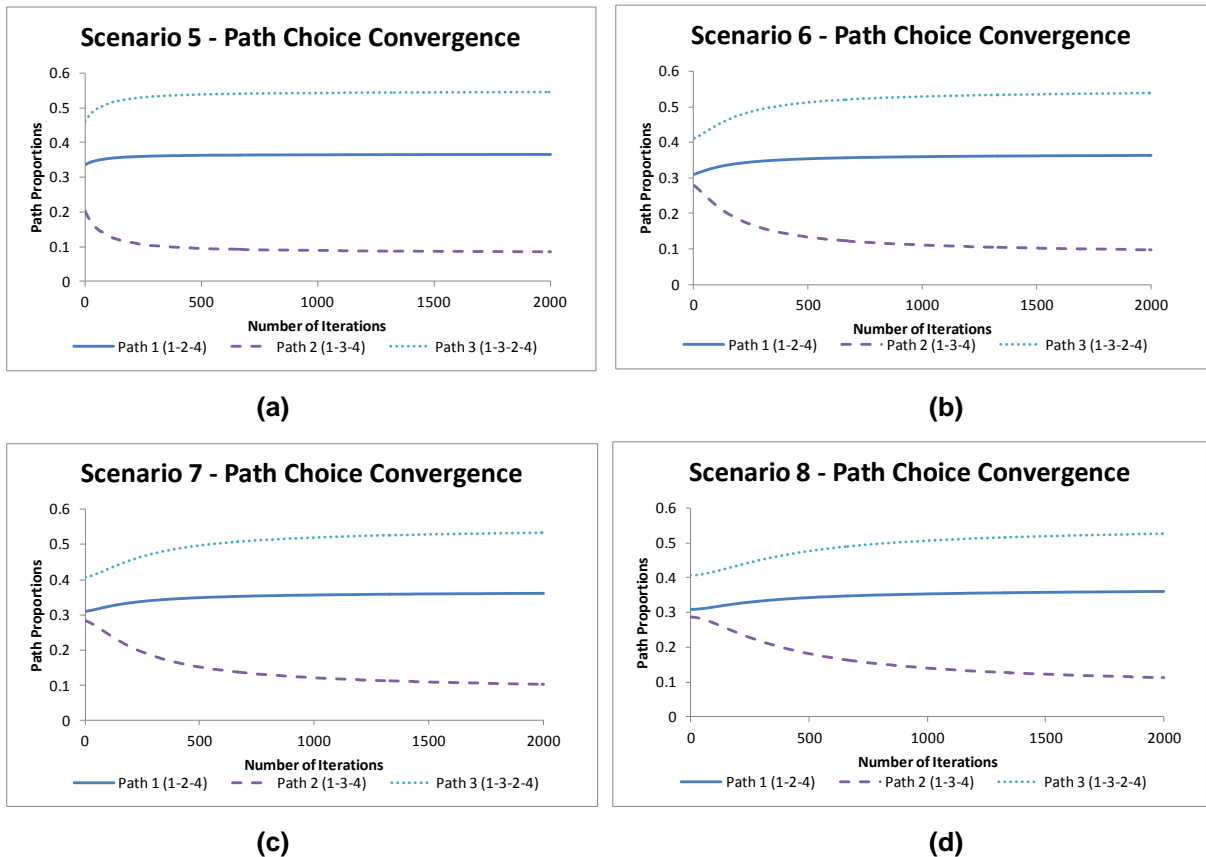
with the system level analysis, results from a subset of scenarios evaluated are presented, which are representative of the trends observed across all the scenarios tested. The paths are hereby referred to as Path 1, 2 and 3, where Path 1 connects nodes 1-2-4, Path 2 connects nodes 1-3-4, and Path 3 connects nodes 1-3-2-4.

4.2.1 Path Choice Convergence: Sensitivity to Initial Perceived Demand Distribution

The convergence of the path proportions over the learning period is illustrated in Figure 4 for four different initial perceived demand distributions. As with the system level analysis, Scenarios 5 through 8 are presented for consistency. The results illustrate that the path proportions converge to within 5% of the actual expected demand for all the scenarios (5 through to 8) within 2000 iterations. Similar to the system level analysis, the convergence rate of the path proportions is sensitive to the accuracy of the initial perception of the users. The results illustrate a clear increased rate of convergence when the initial perceived demand is closer to the actual demand.

Across all the scenarios the path proportions deviate from their initial state. Initially the path proportions for Path 1 ranges between 0.31 and 0.34, Path 2 ranges between 0.20 and 0.29 and Path 3 ranges between 0.41 and 0.46. The differences in the initial proportions are a result of the differences in the initial perceived distributions. As users learn over time the proportions across all scenarios converge to the same values. The changes in path proportions represent a considerable change in link flow over time. In particular the flow on Path 2 has halved over the course of the learning process. These results illustrate the importance of accounting for the learning process in conjunction with new developments that may impact demand, which can have major implications in how we forecast and manage traffic throughout the network. In addition the process can affect infrastructure planning and potentially the ranking of the suitability of infrastructure projects.

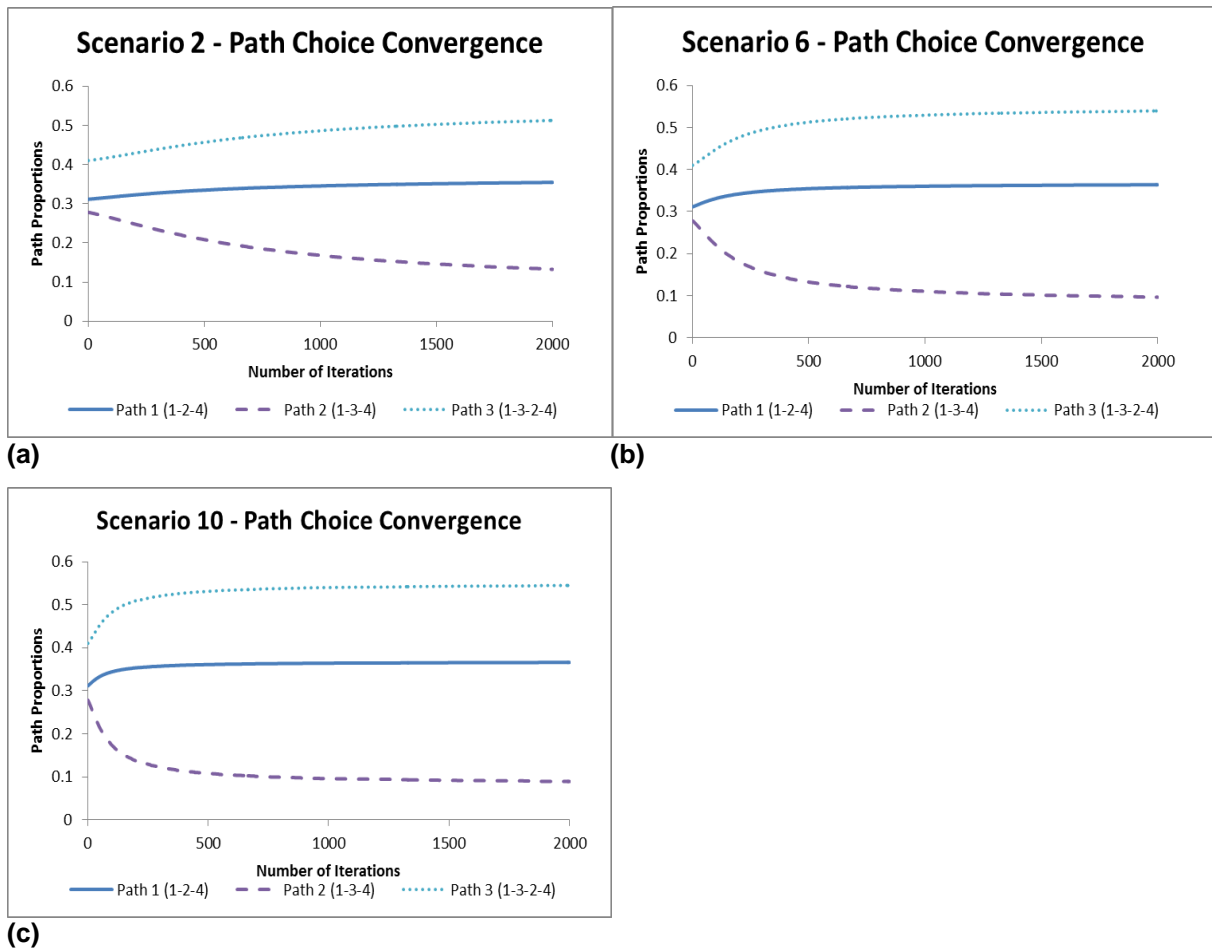
FIGURE 4: Impact of Perceived Demand Distribution on Path Choice for four scenarios a) Scenario 5 b) Scenario 6 c) Scenario 7 d) Scenario 8, corresponding to different initial perceived demand distributions.



4.2.2 Path Choice Convergence: Sensitivity to Precision Variance

The sensitivity of the precision variance on path choice is illustrated in Figure 5. As with the previous sensitivity analysis, Scenario 2, 6, and 10 have been presented, which correspond to a precision variance of 0.1, 0.2 and 0.3, respectively, and a fixed initial perceived demand. The results illustrate the same trends to what was observed in Figure 3. The rate of path choice convergence is significantly affected by the precision variance, with a lower variance corresponding to a slower rate of convergence. As discussed previously, this parameter could represent the familiarity or degree of stubbornness of users of the network. Accordingly further investigation is required to calibrate the true value of the precision variance for a given user group and network, and will be addressed in future work using controlled behavioural experimental procedures.

FIGURE 5: Impact of Precision Variance on Path Choice. The three figures graphs correspond to a variance precision of a) $\tau=0.1$ b) $\tau=0.2$ c) $\tau=0.3$

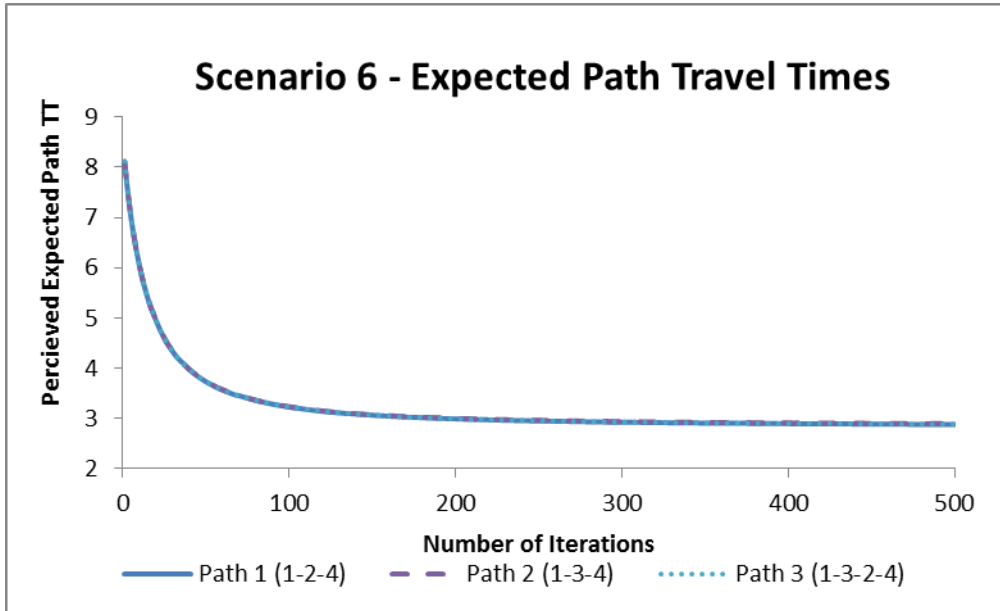


4.2.3 Convergence of Perceived Expected Path Travel Time

Finally, we explore the changes in the perceived expected path travel times by the users over the course of the learning process. The results are depicted in Figure 6. The perceived travel times provide the basis for the users' route choice decisions. Thus, evaluating how these costs change throughout the learning period can provide insight into users' expected route choice. In Figure 6 the expected path travel times for all three paths are shown to overlap. This is consistent with the definition of the StrUE model, for which a Wardropian Equilibrium solution is based on the *expected* path costs, and in the case of L_StrUE, the *perceived expected* path costs. The figure also illustrates that perceived expected path travel times are initially much greater than they are under the actual demand distribution. The

results also illustrate a quick convergence to the correct distribution. For scenario 6, the perceived expected travel times converges in around 100 iterations, to 2.84. This time period is consistent with Figure 5(b), in which the path proportions stabilize after the same number of iterations. The results from this type of path level analysis can be used to reveal how quickly the impact of a new development is learnt by users.

FIGURE 6: Illustration of Convergence of Perceived Expected Path Travel Times



5. Conclusion:

This study proposes a methodological framework which can be used to model the day-to-day learning process of road users after a new development or infrastructure project is in place. The work presented in this paper is novel in two main ways: (i) the application of focus here is the impact of specific new developments on route choice and the immediate adjustment period, versus normal day-to-day conditions and (ii) the Bayesian Inference model is employed to model the learning process within the StrUE assignment model, which is implemented to compute the underlying traffic assignment pattern each day. Numerical analysis was conducted to investigate the sensitivity of the learning process with respect to two main factors, how accurate drivers' initial perception was relative to the actual travel demand distribution, and the impact of the drivers' confidence in their initial perception.

Results illustrated that drivers learned the true demand distribution for all scenarios evaluated. The learning period was shown to be highly dependent on the precision variance, or the drivers' level of confidence in their initial perception. The lowest precision variance, corresponding to a higher confidence level, resulted in a considerably slower convergence process, and longer learning period. In contrast, higher precision variances, representing "new road users" or an "unfamiliar road users", corresponded to much shorter learning periods. Similar trends were evident at the path level and system level. The results from this type of analysis can be used to reveal how quickly the true impact of a new development is learnt by users, and provide insight into users' expected route choice throughout the assessment period.

Future research will address the development of L-StrUE as well as the application into different transport contexts. There is considerable scope to further develop the L-StrUE model. The assumption regarding the equality of the location parameter of the actual demand and perceived demand can be relaxed and different conjugate priors, such as the normal distribution, can be applied to observe any differences in behaviour. Furthermore

controlled behavioural experimental procedures need to be conducted to understand the true value of the precision variance for a given user group and network and used as a calibration tool for the model. These developments will enhance the modelling and understanding of the cognitive learning processes that a user makes whilst travelling.

A learning model such as L_StrUE, has a number of applications in addition to the assessment of changes to infrastructure and the urban environment within a transportation context. L_StrUE can be further developed to understand the impact of major disruptions and disasters to a network. The removal of a link or area of a network will affect the actual demand distribution and perceived demand distribution of the users resulting in a learning process. Another key area where an adaptation of L_StrUE can be applied is within public transit modelling. The impact of learning within a road network can affect the performance and reliability of bus systems consequently impacting the way we value the implementation of these systems. These and further applications will be considered as future research efforts in extending the L_StrUE formulation.

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