Infrastructure Investments under Uncertainty with the Possibility of Retrofit

Theory and Simulations

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Abstract

Investments in large, long-lived, energy-intensive infrastructure investments using fossil fuels increase longer-term energy use and greenhouse gas emissions, unless the plant is shut down early or undergoes costly retrofit later. These investments will depend on expectations of retrofit costs and future energy costs, including energy cost increases from tighter controls on carbon emissions. Simulation analysis shows that the retrofit option can significantly reduce anticipated future energy consumption as of the time of initial investment, and total future energy plus retrofit costs. The more uncertain are the costs, the greater the value of this option. However, the future retrofit option also induces more energy-intensive infrastructure choices, partly offsetting the direct effect of having the option on anticipated energy use. Efficient, forward-looking infrastructure investments have high potential for reducing long-term energy consumption. Particularly if energy prices are expected to rise, however, the potential for reduced energy consumption will be eroded if expectations of energy prices do not include environmental costs or future retrofit possibilities and technologies are not adequately developed.

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Infrastructure Investments under Uncertainty with the Possibility of Retrofit: Theory and Simulations*

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1. Introduction

This paper presents an analytical model framework for choice of energy consumption in relation to energy-demanding infrastructure investments, and simulations to illustrate properties of this framework. An important premise for the analysis is that infrastructure investments, sunk at an initial time of decision, “tie up” energy consumption for a long future period. In our stylized and simple model, time is divided into two discrete periods: an initial period 1, “the present”; and a “future” period 2, which may be much longer. All relevant variables are assumed to be commonly known at the start of the period, for that period.

Several key cost variables in period 2, including energy and environmental costs and certain technological costs (so-called “retrofit” costs, to be explained below), are unknown in period 1, but ex ante statistical distributions, for their period 2 realizations, are assumed to be known (or knowable) at the time when the initial infrastructure investment is sunk in period 1. We assume that the infrastructure persists throughout both periods, but may be (deliberately, and at no additional cost) shut down at the start of period 2. The energy consumption tied to the infrastructure is by assumption based on fossil fuels, at least initially.

The motivation for our analysis is that many types of infrastructure lead to considerable climate policy “inertia”, in that they establish levels of fossil-fuel consumption that may be difficult to reduce later. As is increasingly recognized in the literature, including Ha-Duong et al (1996), Wigley (1996), Ha-Duong (1998), Lecoq et al (1998), and Shalizi and Lecocq (2009), the presence of such an established infrastructure may form a major ex post obstacle to effective mitigation policy, for a long future period, possibly 50-100 years or more. This holds regardless of whether the initial infrastructure investment is “optimal” (in an ex ante sense), or not. The particular problem with infrastructure investment in this context is that costs of mitigation or abatement, to reduce emissions to desirable levels, may, at least in some cases, be very high ex post after the infrastructure has been established.

Below we discuss reasons why the infrastructure investment may turn out not to be optimal. The bias is then typically in the direction of too high fossil energy consumption and carbon emissions. Potential reasons are many and include systematic under-valuation of future energy costs; failures to incorporate true (current and future) social carbon emissions costs; and excessive discounting.

In our analysis we use a very stylized model of infrastructure investment, introducing two potential mechanisms by which the fossil-fuel consumption can be modified “ex post” (in period 2).

The first is to “retrofit” the infrastructure in period 2, at a cost. After a “retrofit”, we assume, infrastructure operation causes no emissions of greenhouse gases (GHGs) from then on. A “retrofit” can be interpreted in several ways. First, infrastructure may be viewed as run without any use of fossil fuels from then on. This is particularly relevant when fossil fuels are replaced by alternative (non-fossil) energy sources, and the use of these sources reduces or eliminates the emissions of GHGs from the “normal” operation of the infrastructure. But this

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1 This of course is a simplification at least relative to some presentations where the distribution of future energy and environmental cost is assumed to be unknown at an initial stage; this can lead to some serious problems of inference as argued e.g by Weitzman (2009). Here, we may use a standard Bayesian approach to justify our position; namely, as the “best initial assessment” of the distribution given our current knowledge. See also Geweke (2001) and Schuster (2004) for formal justifications of this approach.
interpretation is also relevant when applied to cases where the overall energy demand of the infrastructure is (dramatically) reduced. In our stylized model, emissions are also in such cases assumed to be eliminated completely. With either interpretation, the existence of a potential retrofit option in no way implies that retrofit is necessarily optimal; in many cases, exercising the option of retrofit would be prohibitively costly.

Alternatively, a “retrofit” could mean that the consumption of fossil fuels in operating the infrastructure is unchanged, but that the carbon is removed from these fuels (through carbon capture and storage, CCS, or similar technologies or processes). It can however be a more problematic interpretation in our model given that the post-retrofit operating period $T$ is variable, since the retrofit cost is then “periodized” and assumed constant “per time unit” within period 2. When $T$ is variable, retrofit costs with a given distribution $G$ will correspond to a variable total retrofit cost (as it would be proportional to a variable $T$). When the retrofit cost in fact represents a given sunk cost initially in period 2, the $G$ function will need to be amended when $T$ changes.

Both “retrofit” and basic energy/climate cost in period 2 are assumed to be uncertain in period 1, but with initially known joint distribution, assuming that the two costs are non-negatively correlated. Retrofit costs could easily be more uncertain than energy and climate costs, since the nature of future (period 2) retrofit technologies is likely to be unknown at the start of period 1 when initial infrastructure investments must be made.

The second way to avoid energy consumption related to existing infrastructure in period 2 is to abandon it, “close it down”. Since this investment is then rendered worthless, this is a wasteful, and painful, option since the initial infrastructure investment is costly to establish. It can still be attractive and rational ex post, given that energy and/or retrofit costs of continued operation both turn out to be both very high: the lower of the two costs must be higher than the utility value of continued operation. The abandoned infrastructure will then, presumably, be replaced by alternative, less energy-intensive, infrastructure in period 2. The probability of such a simultaneous occurrence is greater when energy and retrofit costs are positively correlated. Our closedown alternative represents a “benchmark” case with zero emissions and energy consumption; and we do not specify any particular replacement alternative.²

We characterize ex ante strategies for establishing energy and emissions intensity associated with the initial infrastructure investment; ex post strategies for retrofitting and operating the infrastructure at a “later” stage (in period 2); and interactions between these strategies. We characterize optimal infrastructure investment, and (important from a climate policy standpoint) identify factors behind inefficient (too energy and emissions intensive) infrastructures being established. We also study whether, and to what extent, an initially high energy intensity level can be modified in later periods through retrofit or closedown, in cases where energy and environmental costs are high.

“Optimal” infrastructure choice is defined for given current prices, and given distributions of future prices. “Optimality” can be established either for a private agent making the infrastructure decisions, or for a social planner; which if these will be invoked in the following will depend on the context. The decision-making agent could be private, but in most cases a public-sector entity (a local or national government). A social planner, taking a

² One possible interpretation of this case is that energy consumption and emissions in the closedown alternative serve as a reference “zero” point, relative to the “business-as-usual” and retrofit alternatives.
national, regional or global perspective, will tend to incorporate prices, costs, discount rates etc. given at the respective (national, regional or global) level, and, we assume, optimally from that particular point of view. A fundamental problem with this, in a climate policy context, is that such a view tends not to be correct, even when formulated at the national level. As a key feature of the GHG emissions control problem, a global view is needed, where the marginal externality cost at the global level is incorporated. Local decision makers are likely not to behave in a globally optimal way, except when international agreements dictate that globally optimal (emissions and energy) prices be applied. Little today indicates that such “optimal” prices will be applied in the near or intermediate future; thus a discrepancy between the ideal, global, social planner and the practical decision maker will be the order of the day. One objective of our analysis is to study how much such a decision maker is likely to deviate from a “socially optimal” decision (from a global point of view).³

Increased availability of retrofit and closedown options affect expected fossil-fuel consumption and GHG emissions over the lifetime of a given infrastructure in two opposing ways. First, they reduce expected fossil-fuel consumption and emissions through the option to avoid such consumption and emissions ex post, in states where emissions and energy costs are particularly high, where the infrastructure can then instead be retrofitted or closed down. On the other hand, an anticipated increased availability of such options serves to increase the chosen energy intensity embedded in the infrastructure when established. A greater availability of the two additional options make it less risky for the decision maker to choose a high initial (fossil-fuel) energy efficiency.

Simulations, in section 3 below, show that a higher variance on retrofit and/or energy costs (for given unconditional expectations of these costs) reduces expected future costs, both in energy terms, and (energy plus retrofit) costs terms. With more uncertainty about both energy and retrofit costs (for given unconditional expectations), the retrofit option is exercised in more cases, and to greater benefit (more states with “gainful” retrofit).⁴ Greater variances are beneficial as they open up for more low-cost alternatives, which are exploited under an optimal ex-post policy.⁵ The gains from greater cost variability are reduced when costs are positively correlated (assumed in some of our simulations).

The distribution of retrofit costs in period 2 depends on technological retrofit possibilities, which in turn are affected by R&D efforts to develop such technologies. While R&D efforts are not modelled explicitly in the paper, we study exogenous shifts in the distribution for such costs. When such cost reductions are correctly anticipated at the time infrastructure investments are made, they lead to increased energy intensity of the initial infrastructure investment choice. Overall expected energy consumption over the lifetime of the project may then either increase or decrease, depending on which of two effects is stronger: this initial effect on infrastructure energy intensity, which results in higher energy consumption in “business-as-usual” states (from the higher energy intensity of the initially established infrastructure); or the lower energy use ex post, due to “business-as-usual” states being fewer and the infrastructure expected to be retrofitted in more states (in period 2 of our model). Simulations, in section 3 and appendix B, show that the latter factor tends to dominate in our model; except possibly in a special case where the initial energy consumption is very sensitive

³ See Strand (2009) for further elaboration on these issues.
⁴ This conclusion holds when the decision maker is risk neutral as assumed in our presentation; it may need qualification under risk aversion.
⁵ This result holds when decision makers are risk neutral, which is assumed here. Under risk aversion, the utility effect of greater uncertainty could here go either way.
to future energy costs (when the utility function for infrastructure services takes a Cobb-Douglas form). Overall expected energy use and climate impacts are reduced with more energy technology R&D, resulting in lower expected retrofit costs. Perhaps equally reasonably, however, (major) reductions in expected retrofit costs will not be fully anticipated at the time infrastructure investments are made (or, at least, are not fully built into these infrastructure decisions). In such cases, reductions in expected infrastructure costs will reduce expected energy costs by more, as they are not accompanied by offsetting effects (in terms of higher energy intensity) for the initially established infrastructure.

Overall expected retrofit costs may in such cases increase or decrease. The factor contributing to a decrease is the very drop in cost. The factor contributing to an increase is the retrofit option being exercised in more states of the world (thus avoiding energy expenditures in more ex post states). Typically here also, the first factor seems to dominate.

More interesting from a policy perspective is what happens in response to an upward shift in the ex ante distribution of period 2 energy costs, here taken to include any climate or other environmental costs resulting from energy use; a key issue in a climate policy context. We study the system’s response to increases in the future cost of carbon emissions and/or energy that may or may be not fully anticipated at the time when major infrastructure investments are made. Our model provides answers to both the following two questions: What is the fossil energy use (and carbon emissions) response, when these future cost increases are fully anticipated, and expected to be incurred? And what is the response when they are not expected to be incurred (as may be the case for some climate-related costs)?

When an increase in expected future energy cost is not anticipated at time of investment, the embedded energy intensity in these investments will be excessive. Our simulations here indicate that when a doubling of the energy/environmental cost can be anticipated, and this is not considered in making the infrastructure investment, it could result in an overall increase in fossil energy consumption by 30-60 percent, even when the ex post response to the energy price increase is optimal. It is here still the case that expected energy use and emissions will be reduced ex post in proportion to the increased use of the retrofit option. Most of this reduction is realized in states with high energy costs, thus leading to (perhaps substantial) expected energy cost savings.

When the increase in future expected energy cost is correctly anticipated at time of infrastructure investment, lifetime expected energy use and emissions are reduced by more than when higher energy costs are not anticipated. The additional factor is that the energy intensity of the initial infrastructure is reduced. We show, mainly through simulations, that the compounded effect of these two factors can be substantial, and that greater uncertainty about energy and retrofit costs adds to this effect.

When an increased future energy and climate cost is not expected to be incurred (but correctly anticipated) by the decision maker (be it a national or local government, or a private party), it has no effect on decisions, neither ex ante nor ex post. This implies policy failures in two different respects: both making the infrastructure excessively energy intensive; and allowing for too few retrofits ex post. When again there is a doubling of the energy price which is not considered, our simulations here show (see table 1 in the final section 4) that the degree to which energy consumption is excessive could be dramatic, and could exceed 200 percent, when compounding the effect of excessively energy-intensive infrastructure ex ante, and excessive energy consumption ex post.
The analytical and quantitative literature dealing with such issues is small. Arthur (1983), David (1992) and Leibowitz and Margulis (1995) provide background by defining and discussing the issue of path dependency and its implications for future actions. The more specific topic of infrastructure choice and its implications for mitigation policy is discussed only recently. Shalizi and Lecocq (2009) provide a discussion of infrastructure costs and constraints which is more applied and intuitive than that provided here. The persistent effects of infrastructure choice on energy consumption and carbon emissions are discussed also by Brueckner (2000), Gusdorf and Hallegatte (2007a,b), and Glaeser and Kahn (2008). In particular, Gusdorf and Hallegatte (2007a) study the energy intensity of urban infrastructure for given population density. They focus in particular on inertia resulting from established urban structure, in response to “low” initial energy prices, which may later rise. They show, through simulations, that a permanent energy price shock leads to a transition period that is long (20 years or more) and painful (with high energy costs, and carbon emissions), but that energy consumption eventually tends to fall toward a substantially lower steady-state level. Glaeser and Kahn (2008) by contrast focus on energy consumption implications of differences in population density, both within urban regions and when comparing urban and rural population patterns. In this context they seek to quantify relationships between energy consumption and spatial patterns of cities in the U.S. They find, in particular, much lower per-capita energy consumption, and carbon emissions, in central cities than in suburbs. This bears on our analysis as it indicates that “compact” infrastructure (as found e.g. in central cities) is less energy demanding than “less compact” (found e.g. in suburbs).

The option to retrofit already established infrastructure, by removing either the initial energy requirement, or the carbon emissions associated with it (via CCS technology or replacing fossil fuels by renewables), reduces the inertia associated with infrastructure. This is a focus in this and two related papers, Strand (2010) and Framstad and Strand (2010). Strand (2010) considers different utility function representations and their implications, in a similar setting. Framstad and Strand (2010) study optimal infrastructure investment when future energy prices follow a continuous stochastic process, where a delayed retrofit decision has a positive option value. Implications of retrofit possibilities and costs are further discussed analytically by Jaccard (1997) and Jaccard and Rivers (2007). The latter paper studies three more specific types of demand-side infrastructure: urban structure; buildings; and equipment. The authors argue, based on simulations (and using a discount rate of 3 percent), that for buildings, and even more for urban structure, it is generally advantageous to make strong considerations for future emissions even when emissions prices start low and increase strongly over time (while this is often not the case for equipment where natural turnover provides sufficient flexibility). Shalizi and Lecocq (2009) provide a broader and more practically oriented discussion, with examples from both energy demand and supply; their overall argument is that energy-intensive infrastructure involving supply is generally more rigid than that involving demand; but sometimes (but not always) more prone to complete retrofit.

Other literature connections need mention. One is the “low-carbon society” and ways to achieve it, treated by many; among others Strachan et al (2008a, b), and Hourcade and Cerassous (2008). This basic idea is, as here that achieving a society with low GHG emissions (necessary for efficiency in the long run) requires a high concern for infrastructure investment design. Another connection is two World Development Reports, the WDR 2003

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6 A very early champion of this line of thinking and discussion was Amory Lovins; see, in particular, Lovins (1977).
(Sustainable Development in a Dynamic World; World Bank (2003)), and the most recent WDR 2010 (Development and Climate Change; World Bank (2009)). In both, “inertia in physical capital” (echoing our analysis of infrastructure) is a main theme.

2. Two-Period Model with Uncertain Retrofit Costs

2.1 The Basic Optimality Problem

Consider a world existing for two “periods”. Infrastructure investment is made at the start of period 1, and can be “retrofitted” at the start of period 2. As long as it is operated and not retrofitted, a given infrastructure gives rise to a given energy consumption per unit of time, determined at the time of initial investment. Energy supply costs and environmental/climate-related costs are uncertain at the time of establishment in period 1, but are revealed at the start of period 2. We assume that when retrofitted, the infrastructure is purged of all fossil-fuel energy content and/or all its carbon emissions. The infrastructure however still provides the same utility services to the public as it did before the retrofit. “Retrofits”, we assume, are not available in period 1: they represent a new technology, developed and available at the start of period 2.

Period 1 has unit length, while T is the “length” of period 2. T can in principle be given two alternative interpretations. First, it could simply be interpreted as the time elapsing during period 2, relative to the initial (unit) period. To invoke this interpretation in the following, it would need to be coupled with an assumption that decision makers choose a zero discount rate. Alternatively, T could embed discounting, in which case it would represent the discounted value of period 2 relative to that of period 1. Under this interpretation, heavier discounting would lead to reduced T for given period length.

We also assume that the infrastructure can in principle be shut down at the start of period 2. Such action will be taken when the total utility of operating the infrastructure is less than the minimum of the energy cost of operation, and the retrofit cost, in period 2.

In period 1, the unit energy cost is \( q_1 \) (given and constant). The policy maker decides on an infrastructure investment with given capital cost \( K \). For simplicity and to focus on other issues than investment size, assume that all relevant infrastructure projects have the same investment cost. Infrastructure type is identified by a given energy intensity \( H \), where we assume that all energy consumption associated with the infrastructure is fixed once the infrastructure is established, and until it is possibly retrofitted. Considering only economically viable projects, we focus on one particular trade-off only: an infrastructure project with higher energy content must give higher immediate utility, but will be more costly to operate (as long as not being

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7 In the model as it otherwise stands, the assumption that a retrofit can be done only at the start of period 2, and not during this period, is no limitation as, we assume, no new information (nor any new or better retrofit technology) will be forthcoming during period 2.

8 More precisely, when unity represents the present discounted value of a current income flow of one dollar throughout period 1, \( T \) would in this case represent the present discounted value of a current income flow of one dollar throughout period 2, as evaluated from the start of period 1.

9 In the continuation, when we say “energy cost”, we mean the combined energy and environmental cost associated with (fossil-fuel) energy use. This would be unproblematic when all environmental costs are charged to energy use in the form of energy taxes and quota prices. It is more problematic when this is not the case; this issue is elaborated in the final section.
retrofitted) due to its greater fossil-fuel energy requirement. Denote the current (per time unit) utility flowing from the infrastructure when being operated by $U(H)$, where $U'(H) > 0$, $U''(H) < 0$. We assume that $U(H)$ is given and constant and the same in both periods (and thus not subject to uncertainty).

Three alternative actions may be chosen in period 2:

1. *No new action, proceed with “business as usual.”* In this case the full energy cost will be incurred in period 2. This is the optimal strategy when the energy cost in period 2 turns out to be lower than both the retrofit cost, and the period-2 utility of the infrastructure.

2. *Retrofitting the infrastructure.* This is the optimal strategy when the retrofit cost in period 2 turns out to be lower than either the energy cost, or the period-2 utility of the infrastructure.

3. *Infrastructure closedown.* This is optimal when environmental and retrofit costs both turn out higher than the period-2 utility of the infrastructure. Closedown is a drastic measure. It will typically require that other infrastructure be supplied, to replace the services lost by project closedown. This is not explicitly modelled here. Implicitly our model embeds such effects, via the absolute value of the utility flow provided by the infrastructure (which should be defined relative to a situation where the utility is missing; thus a “relatively drastic” alternative).

The problem of a decision maker in establishing the infrastructure in period 1 is to select an energy investment intensity $H$ so as to maximize

\[
EW(1) = U(H) - qH + EW(2)
\]

where $E$ is the expectations operator and $W(2)$ is the (optimized) value function associated with the infrastructure in period 2 (embedding the optimal action, among alternatives 1-3 above).

$EW(2)$ embeds the decision maker’s optimal responses at the start of period 2 (assuming that no further changes occur during period 2). Define $F(q,y) = F_{qy}$ as the (continuous ex ante, when viewed from period 1) cumulative bivariate distribution over $q$ and $y$ levels to be realized in period 2, with support $[0, q_M] \times [0, y_M]$, where $q_M$ and $y_M$ could be large.\(^{10}\) Possibly, the marginal distribution $F_q$ for period 2 is shifted up by increased emissions in period 1. Here, $y$ represents retrofit costs (per unit of energy capacity to be retrofitted) in period 2. Retrofit costs cannot be negative, but could in principle be small in period 2, depending on the technology available for substituting out the fossil-fuel energy consumption or purging carbon from fossil fuels at that time. We assume that an infrastructure project, after a retrofit, incurs no energy costs nor any other current costs in period 2, apart from the retrofit cost itself (which in the model is “periodized” in the same way as energy cost).\(^ {11}\) In the

\(^{10}\) In simulations below we assume that $F$ is bivariate log-normal, in which case $F$ is not bounded above (it is however “thin-tailed”).

\(^{11}\) Alternatively, the retrofit cost could be interpreted to include some energy cost. This is unproblematic as long as the retrofit cost can be periodized.
analytical presentation, period 2 realizations of energy cost and retrofit cost are not assumed to be independent.  

Consider the choice between the three alternatives lines of action 1) – 3) in period 2. We start with action 3), project closedown. Define total utility per energy unit for installed infrastructure by \( U(H_1)/H_1 = y^* \), where \( H_1 \) is energy intensity associated with the infrastructure investment chosen in period 1. Action 3) will then be chosen when the cost per unit of energy \( q \), and the retrofit cost \( y \), both exceed \( y^* \). The probability of this event, when viewed from period 1, is

\[
P(3) = \int_{q=y^*}^{\infty} \int_{y=y^*}^{\infty} f(q,y) dq dy.
\]

Assume \( 0 < y^* < \min \{ q^M, y^M \} \), and \( 0 < F(y^*, y^*) < 1 \), implying \( P(3) > 0 \).

The probability \( P(1) \) of inaction (action 1), is given by the following expression:

\[
P(1) = \int_{y=q}^{\infty} \int_{q=0}^{y^*} f(q,y) dq dy.
\]

The probability \( P(2) \) of retrofit (action 2) is complementary (equal to \( 1 - P(1) - P(3) \)), but can also be found in a similar way as \( P(1) \), as follows:

\[
P(2) = \int_{q=y}^{\infty} \int_{y=0}^{y^*} f(q,y) dq dy.
\]

Note that we can switch the order of any double integrals with an application of Fubini’s theorem. Define the following (unconditional) expected unit cost variables in period 2, related to energy cost and retrofit cost respectively, by

\[
E_q(2) = \int_{q=0}^{\infty} q \int_{y=0}^{\infty} f(q,y) dy dq
\]

\[
E_y(2) = \int_{y=0}^{\infty} y \int_{q=0}^{\infty} f(q,y) dq dy.
\]

These are costs that would be realized given that, in the first place, “business as usual” energy use is applied in all states in period 2; and in the second place, retrofit is applied in all states in

12 Correlation of energy and retrofit costs is often realistic. Such correlation could in principle be either positive or negative. Negative cost correlation could occur when the energy cost in period 2 is anticipated during the period of R&D efforts to develop new retrofit technologies. A high anticipated energy cost could then make the development of retrofit technologies more urgent, and more effort expended for that purpose. This endogenous process could then lead to negatively correlated costs. On the other hand, common drivers may affect both costs in the same direction. This is relevant e.g. when energy cost is correlated with general production cost; when a retrofit involves some use of fossil energy; or when the subsequent use of renewable energy whose marginal production cost is positively correlated with the cost of fossil fuels. In such cases the two cost variables would tend to be positively correlated. This corresponds to our main assumption, for the simulations below.

13 See e.g Royden (1988).
period 2. These are not realized costs and instead merely benchmarks (both excessive), for two main reasons: the lower of the cost alternative is applied given that one of them is applied; and in some states the third alternative (closedown) implies lower costs.

Considering now actual realized costs, the expected “per time unit” period 2 energy and retrofit cost as viewed from period 1, given an optimal strategy for period 2, are respectively

\[
E[CH(2)] = \left( \int_{q=0}^{y} \int_{y=q}^{\infty} f(q,y)dydq \right) H_1 \equiv Ech(2)H_1 ,
\]

\[
E[CR(2)] = \left( \int_{y=0}^{\infty} \int_{q=y}^{y} f(q,y)dqdy \right) H_1 \equiv Ecr(2)H_1 .
\]

\(E[CH(2)]\) expresses energy costs per “time unit” in period 2, while \(E[CR(2)]\) similarly expresses retrofit costs when similarly periodized (counted per period unit), under an optimal decision rule for ex post factor choice (“normal operation”; retrofit; or closedown). \(H_1\) denotes (per-period) energy consumption associated with the infrastructure as established in period 1. Define \(E[C(2)] = E[CH(2)] + E[CR(2)]\), as well as \(E(2) = E[C(2)]/H_1\). In particular, we must then have

\[Ec(2) < (1-P(3))\min\{Eq(2), Ey(2)\},\]

as long as all three ex post policy alternatives: “business as usual”, retrofit, and closedown, are actually exercised in period 2.

A factor not apparent from this derivation is that two alternative interpretations of retrofit costs are likely to have somewhat different implications for the analysis. The first is to view retrofitting simply as replacing a fossil fuel with a non-fossil fuel which gives rise to no GHG emissions. Retrofit costs are then incurred currently in the same way as regular (fossil) fuel costs. Under the second interpretation, retrofit costs represent an investment to remove the fuel need tied to the infrastructure, or the emissions associated with the fossil fuel. In (8), this implies that \(E[CR(2)]\) must be “periodized” (given that \(T\) is different from unity), and spread evenly across \(T\) time units in period 2.

Expected (discounted) net utility from the infrastructure when operated in period 2 is denoted \(EW(2)\), and equals the gross utility of the infrastructure, \(TU(H_1) = Ty*H_1\), in states where it is not closed down in period 2 (thus with probability \(1-P(3)\)), minus total combined expected energy and retrofit costs, \(T(E[C(2)]) = T\{E[CH(2)] + E[CR(2)]\}\), over states where the infrastructure is operated (without, or with, retrofitting). We then have

\[
EW(2) = \{y*[1 - P(3)]H_1 - E[CH(2)] - E[CR(2)]\} T .
\]

The first-period decision problem is formulated as maximizing the expected utility of the infrastructure investment in period 1, considering an optimal strategy in period 2. Define

\[
EW(1) = U(H_1) - q_i H_1 + EW(2) = (y* - q_i)H_1 + EW(2).
\]
Assume for now that the distribution of period 2 energy (including environmental) costs is exogenous (and not affected by emissions arising from the infrastructure). The solution to the maximization problem in period 1 takes the form

\[
\frac{dEW(1)}{dH_1} = U'(H_1) - q_1 + \frac{EW(2)}{H_1} + [U'(H_1) - y^*] \frac{d\left(\frac{EW(2)}{H_1}\right)}{dy^*}.
\]

From (9) we find

\[
\frac{d\left(\frac{EW(2)}{H_1}\right)}{dy^*} = [1 - P(3)]T.
\]

Using the definition of \(EW(2)\), and setting the derivative in (11) equal to zero, we find the following implicit expression for the optimal energy intensity of the infrastructure:

\[
U'(H_1) = \frac{q_1 + \frac{E(CH(2) + E(CR(2))}{H_1}}{1 + [1 - P(3)]T}
\]

The optimal energy intensity is chosen according to average energy cost in operating the infrastructure, over its expected period of operation. Perhaps surprisingly, the extent of the operation period as such is not very important for the chosen intensity.\(^{14}\) For e.g. a hypothetical case where the infrastructure is shut down in period 2 for certainty; the energy intensity would be determined simply by \(U'(H_1) = q_1\). When average energy cost in period 2 exceeds \(q_1\), \(U'(H_1) > q_1\), and \(H_1\) lower (since the required utility return per unit of infrastructure must be higher which requires a lower infrastructure mass).

\(^{14}\) This result is closely related to our initial assumption, that the size of the infrastructure investment is exogenously given.
For $y^* (= U(H_1)/H_1)$ we find

\[
\frac{dy^*}{dH_1} \equiv y''_u^* = -\frac{1}{H_1} (y^* - U') .
\]

Here $y^* - U'$ must be positive and more so the more curved the utility function is in the neighborhood of $H_1$. Thus we can expect $y''_u^* < 0$.\(^{15}\) This implies that a more energy-demanding infrastructure will have a lower threshold for operation, and will be “closed down” (or rather, replaced) ex post in more cases in period 2, when energy and retrofit costs increase. This may appear reasonable; remember that in our model all infrastructure projects are considered to be “equally large” in the sense of requiring the same initial investment cost. What distinguishes projects is the ex post energy requirement for their operation (or put otherwise, their energy intensity).

Appendix A1-A2 deals with comparative statics for the above model, when there are period 2 shifts in the distribution of energy costs (A1), and in the distribution of retrofit costs (A2), for a simplified case with independent costs.

### 2.2 Analytical Specification of the U Function

We will in this sub-section consider the main class of the utility function specification, $U(H_1)$, that will be applied in the simulations in section 3 below. Our basic assumption is that $U(H_1)$ belongs to the class of constant relative risk aversion utility (CRRA) functions. The (vNM) utility function then takes the following general form:

\[
U(H_1) = A \frac{H_1^{1-\rho}}{1-\rho} + K,
\]

where $A$ and $\rho$ are positive parameters, and $K$ is a (non-negative) scaling parameter ensuring that $U$ only takes non-negative values on its relevant ranges.\(^{16}\) The (Arrow-Pratt) measure of relative risk aversion is given by $\rho$. Define $\left[\frac{q + Ec(2)T}{1 + (1-P(3))T}\right] = Q$ as the expected total unit cost of operating the infrastructure over its lifetime, per expected time unit of operation (where $T$ is the number of time units within period 2, and $1-P(3)$ is the probability of operation in period 2), and per unit of established infrastructure, $H_1$. We have defined $Ec(2) = \frac{E(C(2))}{H_1}$ (as the average expected realized cost per time unit in period 2, per unit of invested $H_1$), which from (7)-(8) is constant for given distributions of $q$ and $y$. Maximizing ex ante net utility associated with the initial infrastructure investment, given by $U(H_1) - Q$, we derive the first-order condition

\[
AH_1^{1-\rho} = Q
\]

Here all magnitudes except $H_1$ can be viewed as constants. Consider the effect on $H_1$ when the cost variable $Q$ changes. This is found as

\(^{15}\) Another way of interpreting this can be found by considering that $y^*$ is an expression of the average utility of infrastructure per unit of $H_1$, $y''_u^* < 0$ then simply expresses that average utility of infrastructure must exceed its marginal utility at the point of indifference between operation and closedown.

\(^{16}\) This issue is relevant only for case c) below, where $U$ would otherwise take only negative values.
From (17), the “elasticity of demand for energy intensity”, $\varepsilon_H$, is found as

$$\frac{dH_1}{dQ} = -\frac{1}{\rho} \frac{H_1}{Q}$$

Thus the energy intensity demand elasticity with respect to total operating cost per unit of established infrastructure (but where the initial infrastructure cost is not included) is a constant. All possible values of $\rho$ from zero to infinity are allowed by this formulation. The effect of an increase in $Q$ on total expected costs (for energy and retrofit combined) is

$$\frac{d(H_1Q)}{dQ} = H_1 + Q \frac{dH_1}{dQ} = \frac{\rho - 1}{\rho} H_1.$$ 

We can now easily distinguish between three distinct classes of cases for (15), namely

a) $0 < \rho < 1$: $U$ takes a Cobb-Douglas form. Then $d(H_1Q)/dQ < 0$. Overall expected ex ante (total) operating costs, including the response of the initial energy intensity (and in terms of both expected energy costs, and expected retrofit costs), are then reduced in response to an increase in expected ex ante unit cost. The reason is that the energy intensity chosen in response to a change in the cost variable is then reduced relatively more than the change in unit costs.

b) $\rho = 1$: $U$ takes a logarithmic form. Then $d(H_1Q)/dQ = 0$. Overall expected ex ante (total) cost, including the response of the initial energy intensity, is constant in response when expected ex ante unit operating cost increases. The energy intensity is reduced in response to a change in the cost variable, so as to exactly offset the increase in unit expected ex ante cost.

c) $\rho > 1$: $U$ is exponential. Then $d(H_1Q)/dQ > 0$. Overall expected ex ante operating cost, including the response of the initial energy intensity, here increases in response to an increase in expected ex ante unit operating cost. The energy intensity is reduced but by relatively less than the change in energy costs.

Under this class of utility functions we have the following general expressions for $y^*$ (expressing the ex post marginal cost beyond which the infrastructure will be shut down in period 2), and its change with $H_1$:

$$y^* = \frac{A}{1 - \rho} H_1^{-\rho} + KH_1^{-1}$$

17 Remember here our initial assumption, that all possible infrastructure projects have the same given initial investment cost; infrastructures differ only in their dimensioning of $H_1$, and their utility (which is higher for higher $H_1$).

18 Note also that the utility function in this case takes only negative values for $K = 0$ in (15), and tends to minus infinity as $H_1$ tends to zero. A meaningful application of the model to this case (in particular, a meaningful definition of the ‘closedown” limit operation price $y^*$) would require $K > 0$ and that we restrict attention to $H$ levels.
As noted, under case c) one would here need to impose a minimum positive value of $K$.

The CRRA class, embedding all cases a) - c) where the coefficient of relative risk aversion is allowed to vary from zero to infinity, can be viewed as sufficiently wide for our purposes. In particular, it embeds all relevant possible values of the (crucial) elasticity of demand for energy intensity in infrastructure with respect to expected operating cost. The only effective constraint on this class is the constancy of this elasticity, for any chosen value of $\rho$.

2.3 Optimality, Deviations from Optimality, and the Relation to This Analysis

“Optimality” will here be defined alternately by a (global) social planner, or by the local decision maker choosing the initial infrastructure investment (who is typically not a global planner). It is here useful to identify different sources of deviations between the two concepts, with resulting inefficiency in deciding the initial infrastructure investment, when decisions are otherwise “optimal” (from the point of view of the decision maker). We next wish to identify implications of “non-optimal” choices by the actual (local) decision maker, or choices made under incorrect information. We distinguish between the following five issues (spelled out in more detail in Strand (2010)). This classification is particularly useful for interpreting the simulation results in section 3 below.

A) The initially expected distribution of energy costs, relevant for making current infrastructure decisions, is lower than (or more precisely, down-shifted relative to) the true (correct) distribution. This could occur e.g. when the infrastructure decision is based on an expectation of something close to current energy cost on average in period 2, while the correct distribution implies higher average energy costs. In this case we may expect the infrastructure energy intensity to be chosen at a too high level, and fossil-fuel consumption in period 1 to be excessive. The period 2 realized expected energy consumption is however not obviously higher than optimal in consequence. This is because the closedown and/or retrofit options could then in response be exercised in more cases.

B) The distribution of energy costs facing the policy maker is correctly anticipated, but is down-shifted relative to the “optimal” energy cost distribution. This is relevant whenever the authorities, in the economy in question, implement emissions prices that are lower than “globally correct” prices. This case is highly relevant: as of today, hardly any country implements what most analysis would agree are “globally correct” emissions prices; nor seems to be willing to do so in the foreseeable future. In this case we can expect, unambiguously, excessive fossil-fuel consumption in both periods.

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19 We argue that this could occur even in cases where the entity making the infrastructure decisions would face a “true” energy/environmental cost. One such case is where the administrative procedure for making public investments involves incorporation of future costs and benefits for a limited period (say, 20 years), while a correct investment decision would require a much longer period (say, 50 years or more). An increasing energy price (beyond the 20 year term) would add to the bias involved in such investment.

20 In our simulations below we however do not allow for this possibility at least not with respect to retrofit costs which are throughout assumed to be proportional to energy costs, ex post.
C) Future retrofit costs are incorrectly anticipated. In particular, with a too optimistic view, the anticipated retrofit cost distribution will be lower than the correct distribution. This will bias the infrastructure decision in the direction of excessive energy intensity for the initially established infrastructure. On the other hand, when the distribution of future energy costs is too optimistic, the retrofit cost distribution will be less important for the energy intensity decision, since the prior expectation is then that retrofits are necessary in fewer cases. Instead of “technology optimism” one could however have “technology pessimism” (a too pessimistic view of the distribution of future retrofit costs), which would tend in the opposite direction.

D) Future retrofit costs are correctly anticipated, but are higher than socially optimal costs. Such a case is where the R&D effort in developing new energy technologies is suboptimal, including those for retrofits. Overall implementation costs (in particular, minimum of energy and retrofit costs) would then be shifted upward in period 2. Given correct perceptions in period 1, the choice of energy intensity of the infrastructure is in response lower than optimal (efficient energy intensity is greater than the one chosen), while the probability of ex post business-as-usual operation of the infrastructure is higher than optimal (as there are more states where the energy cost is below retrofit cost). This follows straightforwardly from the fact that the business-as-usual alternative will be chosen in more cases in period 2, and the retrofit alternative in fewer cases. With no other distortions, the energy intensity of the initially chosen infrastructure would then in fact be suboptimal. On balance, the second factor is likely to dominate, leading to socially excessive energy consumption in period 2.

E) The policy-relevant value of $T$, call it $T_1$, is less than the optimal value, call it $T_0$. Reasons for this may be either excessive discounting (for the case where $T$ is interpreted as a discounted value), or that the initial policy decision undervalues the length of period 2. Since $T$, as noted at the start of this section, can be interpreted as a discounted value of period 2 relative to period 1, we may have a discrepancy between the “socially correct” value $T_0$ and the value $T_1$ used when deciding on $H_1$. $T_1 < T_0$ could then reflect excessive discounting. When average costs per operation period are greater in period 2 than in period 1 (as might be expected), this leads to a lower average operation cost for the infrastructure, and a more energy-intensive infrastructure.

In four of these cases (all except D), the overall expected fossil-fuel energy consumption (and GHG emissions) over the potential lifetime of the infrastructure would tend to be excessive from a social point of view, in the sense that it is higher than the expected fossil-fuel consumption and emissions for the case where all global externalities are optimally considered and anticipated.\(^{21}\)

\(^{21}\) One might in addition argue that case D is less likely in practice: when actually realized retrofit costs tend to be higher than their socially optimal level, this will, probably, tend not to be anticipated. If not, the end result will be excessively energy intensive infrastructure, and overall excessive emissions.
3. Simulations

We will in this section illustrate properties of our model through four sets of simulations: 1) ex ante expected probability of energy use or retrofit in period 2; 2) ex ante expected energy and retrofit costs in period 2; 3) energy intensity of the infrastructure chosen in period 1; and 4) overall expected energy use, which combines the energy intensity of infrastructure (3) with the probability of subsequent energy use (1). Under our (we think, reasonable) numerical cases the closedown option turns out to play a very small role for the overall results, and will not be emphasized in the following; but must still be kept in mind. All simulations are made under the assumption that energy and retrofit costs are jointly log-normally distributed, and calculations are done using the scientific program Matlab. Most of the simulations for expected energy cost in period 2 (section 3.2) depart directly from the expected cost expressions (7), and (8) for expected retrofit cost in period 2. For probabilities of using energy and retrofitting respectively (section 3.1) we use (3) and (4), and for energy intensity of infrastructure (section 3.3), we use (13). For expected overall energy use section 3.4), we apply the solution value for \(H_1\) together with the ex ante probability of energy use, (3).

In all cases where nothing is otherwise stated, the (unconditionally) expected energy and retrofit cost (per unit of energy use and retrofit investment respectively) are both kept constant, at levels \(E(q(2)) = 2\), and \(E(y(2)) = 3\). \(E(q(2))\) is the unconditionally expected energy/environmental cost in period 2; it would be the actual expected cost given no retrofit or closedown. A similar interpretation holds for \(E(y(2))\).) While both energy and retrofit costs are uncertain, ex ante energy costs are thus generally lower in expectation. The distributions of energy and retrofit costs are both assumed to be jointly log-normal, and either independent, or have a positive correlation coefficient of 0.5. In sections 3.3-3.4, simulations are done for all variants a) – c) of the utility function (15).

3.1 Ex Ante Probability of Energy Use or Retrofit in Period 2

This first set of simulations considers implications of various parametric changes in the model, for the expected value of (per-period) energy costs during period 2. It is useful to first study period 2 behavior; this will be a benchmark for subsequent analysis of period 1 investment and overall energy use, in sections 3.3-3.4 below.

Figure 1 describes the probability that the firm in period 2 will continue using energy at the level initially established in period 1, given a parametric change in expected energy cost, for different degrees of uncertainty about both energy and retrofit cost.23 In all examples used in figure 1, \(E_y = 3\). We see, as should be obvious, that a low (high) \(q\) implies a high (low)

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22 Two other sets of assumptions in the following also potentially be attacked. One is the log-normal distribution assumption. Log-normality is however a rather robust assumption in this context; see e.g. Schuster (1984). The other is our assumption on the utility function of energy intensity in infrastructure, where we assume a set of functions parameterized by the demand elasticity with respect to price (thus constant for a given specified function; the CRRA class of functions). While this general class of functions can be viewed as robust, criticism can be raised in particular to the Cobb-Douglas and log specifications; see below.

23 Note that the probability of closedown is also accounted for in these calculations, but its probability turns out to be too small to matter for the results. (Throughout, the value of continued use is set at 10, implying that this option will be exercised only when \(\min(q, y) > 10\).)
probability of energy use. Also, this tendency is greater when variances are smaller. This is because, with greater variances, there will be more attractive options on average, to substitute out one variable for the other; and thus a smaller propensity to rely only on the factor that is less expensive in expectation (which is energy for $Eq \leq 3$, and retrofit for $Eq > 3$).

**Figure 1: Probabilities of energy use and retrofit as functions of unit energy cost, for independent costs**

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24 While this is shown in the figures only for simultaneous changes in both variances, the same basic result holds when only one of the variances at the time is changed.
Figure 2: Probabilities of energy use and retrofit as functions of unit retrofit cost, for independent costs

Figure 2 above similarly tells us how these probabilities change when expected energy cost is kept constant and expected retrofit cost instead changes parametrically. A very similar pattern emerges except that it is, of course, the probability of energy use that increases when the expected value increases: the two figures are, in important ways, mirror images of one another.

In appendix B (figures 1.1-1.2) we consider similar calculations as those shown in figures 1-2, except that the distributions for q and y are assumed to be positively correlated, with correlation coefficient 0.5 (and not independent as here). We then find the distributions to be quite similar, with the (small) change that \( P(I) \) is higher for low q, and lower for high q (in figure 1.1). This is because positive cost correlation leaves less scope for gainful substitution, of a cheaper alternative for a more costly, ex post.

The results in the appendix also show that the probability of using energy increases as the variance of energy costs increases; and is reduced (and thus the probability of retrofitting increased) when the variance of retrofit cost increases. This result follows because energy costs are lower than retrofit costs in expectation; increased (reduced) uncertainty of retrofit costs will lead to greater (smaller) probability that energy use is substituted out through a retrofit.
Figure 3: Probabilities of energy use and retrofit, different degrees of uncertainty, as function of degree of cost correlation

Figure 3 above describes effects of a parametric increase in the correlation coefficient for the jointly lognormal distribution of energy and retrofit costs, going from zero to unity, for the case of $Eq = 2$, and $Ey = 3$. We see that a more correlated cost structure generally implies that the probability of energy use increases. This is intuitively reasonable as, in our example, with perfect correlation we would have $q < y$ always and thus energy use always preferred. With less than perfect correlation, there will exist states where $q > y$ and thus retrofit instead used; and more such states when variances are higher. We however see, for the examples simulated, that in no event is the probability of energy use ($P(I)$) below 0.8.

3.2 Ex Ante Expected Energy and Retrofit Costs in Period 2

We now consider simulations of expected costs in period 2, as viewed from period 1 and where $Ey = 3$ (figure 4), and $Eq = 2$ (figure 5). Figure 4 considers parametric changes in $Eq$ under these assumptions, while figure 5 considers similar parametric changes in $Ey$. Interpreting the figures requires some care. Note that, as $Eq$ in figure 4 is increased parametrically from values below 3, to values above 3, energy ceases to be the more economical alternative on the average. Thus, these two curves always cross at $Eq = 3$ (in figure 4), and $Ey = 2$ (in figure 5). Similarly in figure 5, as $Ey$ is reduced below 2, retrofit there takes over as the more efficient alternative on the average. Also, total expected cost will always be an increasing function of both $Eq$ (figure 4) and $Ey$ (figure 5); and the highest value the total expected cost curve can attain is 3 (figure 4) and 2 (figure 5). We see that total expected cost is kept well under these levels, and more so for higher variances. This is again intuitive: high variances give great option for ex post cost minimization thus lowering average expected cost. We also see that, for the high-variance alternative in figure 4 ($var(q) = var(y) =$
conditional expected energy cost keeps increasing well beyond the crossing point 3 (as there are “sufficiently many” states in which energy costs are chosen, so that the cost-increasing effect of $Eq$ going up dominates the overall expected cost picture).

**Figure 4:** Ex ante expected energy/retrofit costs in period 2 as function of unit energy costs, for different variances, independent costs

**Figure 5:** Ex ante expected energy/retrofit costs in period 2 as function of unit retrofit costs, for different variances, independent costs
Figures in appendix B describe how conditional expected energy, retrofit and total costs vary with changes in \( \text{var}(q) \) and \( \text{var}(y) \). Under certainty no retrofits would be incurred in period 2 (since then by assumption \( y = 3 \), higher than \( q = 2 \)). Under uncertainty, additional options open up, as particularly high (energy, and retrofit) costs can be avoided, and cases with low costs implemented. As a result, overall conditional costs will be reduced, and more so with greater uncertainty, represented here by the variances of \( q \) and \( y \). In figure 5, this feature is seen to hold for partial increases in both variances. In particular, when both variances are about 2.5, approximately half of (unconditional) energy cost is avoided, while half as much is added in the form of retrofit cost. The total overall factor cost saving is then about one fourth. Note also, as a general feature of the results from the simulations, that the factor with the lower unconditional expected cost has the higher conditional expected cost. The reason is, obviously, that when the unconditional expectation is lower, the respective alternative will be applied in more cases (and the opposite alternative in fewer cases).

Figure 6 below describes how expected costs (for energy, retrofit and in total) vary with the degree of correlation between the two cost variables, \( q \) and \( y \). All values are here as fraction of maximal expected cost (which equals 2, the minimal of \( E_q \) and \( E_y \)). We see that some cost avoidance is possible (in the sense that overall costs drop below unity in the figures), and more so for low correlation.

**Figure 6: Ex ante expected energy/retrofit costs in period 2 as fractions of maximal expected cost (=2), under different degrees of cost uncertainty, and as function of degree of cost correlation**
3.3 The Choice of Infrastructure Investment

We now simulate the initial infrastructure investment decision, $H_1$, as given in (13) (jointly with factors determining the joint distribution of $q$ and $y$). The reported simulations all assume $T = 5$. We are here thus assuming that period 2 is 5 times as long as period 1 (in expected discounted value terms). Throughout we assume that the probabilities of period 2 action are not affected by $H_1$. The number of time periods is however found to affect $H_1$ (the balance between periods 1 and 2 then changes, and expected costs are generally different in the two periods).

The form of the utility function $U(H_1)$, going into (13), here matters greatly. Figures 7-8 report such results for our three main alternative utility function specifications (all based on CRRA assumptions): a) The Cobb-Douglas case (with exponent $\rho = 0.5$). In this case the elasticity of $H_1$ with respect to total expected ex post cost (after investment is sunk) is $\varepsilon_H = 1/\rho = 2$. b) The log-linear case, in which $\varepsilon_H = 1$. c) The exponential specification exponent 0.5; here we assume $\rho = 1.5$, and thus $\varepsilon_H = 2/3$. Among these specifications, the exponential function provides the least sensitive choice of $H_1$ in response to an increase in $Eq$; and the Cobb-Douglas function the most sensitive, with the log-linear specification as an intermediate case. This follows naturally from section 2.2 above, where we showed that in the log-linear case, an increase in ex ante unit cost (combining energy and retrofit costs) leaves total ex ante cost constant; the reason is that a reduction in $H_1$ exactly offsets the expected unit cost increase in this particular case. In the Cobb-Douglas case the offsetting change in $H_1$ is greater; and in the exponential case, smaller.

While our simulations thus cover a range of possible cases, the realism of some of the alternatives may be questioned. In particular, for some alternative specifications it might be argued that the scope for change in $H_1$ is, perhaps, too great. In particular, under the Cobb-Douglas specification this scope may appear excessive relative to what is seen empirically in most cases. The exponential specification (with $\varepsilon_H = 2/3$ under our parametric example) may then perhaps seem as the most realistic case. This is useful to have in mind in interpreting our numbers in the following.

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25 Conceivably, there may be such effects, which will in case modify our conclusions somewhat; see section 4 below for a discussion.
Figure 7: Energy intensity of infrastructure investment as function of expected energy cost in period 2, for different utility functions and variances, non-correlated costs

Figure 8: Energy intensity of infrastructure investment as function of expected retrofit cost in period 2, for different utility functions, variances, non-correlated costs
The chosen energy intensity is in all cases a decreasing function of both $Eq$ (figure 7), and of $Ey$ (figure 8) with the greatest rates of reduction in the Cobb-Douglas case, and the smallest in the exponential case. In the intermediate logarithmic case, the slope is similar in the three variance cases, but the level of $H_1$ (for any given $Eq$ in figure 7) is higher with higher variances. This is due to conditional expected costs being lower with higher variances. Such anticipated future cost savings are in the Cobb-Douglas case exactly counteracted by a higher energy intensity of the infrastructure. This effect is magnified in the Cobb-Douglas case, and weakened in the exponential case. In the (perhaps more realistic) exponential case the response of infrastructure energy intensity to cost increases is smaller.

**Figure 9: Energy intensity of infrastructure investment for different degrees of uncertainty in period 2, as function of degree of cost correlation**

Figure 9 above describes effects of increased cost correlation on $H_1$, for $Eq = 2$ and $Ey = 3$. In all cases there is a strong tendency for $H_1$ to fall with greater cost correlation, which gives less room for ex post cost reductions. Utility function levels are here less meaningful (and need to be calibrated); only relative levels (along the different curves) are. We see that the scope for cost avoidance is not dramatic: in the log and exponential cases expected costs are reduced by about 10 percent when the cost correlation is reduced from 0.8 to 0.

### 3.4 Ex Ante Expected Energy Consumption over the Project Lifetime

In this final sub-section containing simulations, we compute the (per-period) ex ante expected energy consumption over the project’s lifetime, found by multiplying the expected per-period energy consumption, where period 2 consumption is found by multiplying the probability of energy use by the intensity of the infrastructure. As before we assume $T=5$. The infrastructure
is operated in a “business as usual” way (and energy used) with probability 1 in period 1, and with probability $P(I)$ in period 2.

Energy consumption is here strongly decreasing in $Eq$, and, typically, more strongly so than what was found for $H_L$ in the section above. The reason is that ex ante energy consumption depicted in figure 10 compounds two separate effects on energy consumption, and both go in the same direction: first, $H_L$ (the energy intensity of infrastructure at establishment) is reduced; and secondly, the probability of using energy ex post, $P(I)$, is reduced. This provides for a, potentially large, reduction in energy consumption in response to a (correctly and rationally anticipated) future increase in expected energy costs, given that the infrastructure decision, fully and rationally, takes such future cost increases into consideration.

The energy consumption response in figure 10 is “efficient” in the sense that the infrastructure decision is fully considered, and fully rational using “correct” values for future energy costs; and embedding the future retrofit possibilities optimally. In such cases, we can expect energy use to be highly responsive to changes in the level of these future expected costs.

**Figure 10: Expected per-period energy consumption over the project’s lifetime, as function of expected energy cost in period 2, different utility functions, independent costs**

This is of course a tall order in terms of being a description of reality. When instead infrastructure is not (at all) responsive to future expected costs, the simulations in section 3.1 above (ex post probability of energy use only) represent the only way in which energy consumption can respond to changes in energy prices (which, we should recall, by assumption contain all relevant climate and environmental costs). In addition, the retrofit possibilities considered here can be viewed as representing some “optimal” development of such
possibilities; when these are not forthcoming the ex post probability $P(1)$ cannot change (much); and energy use will be essentially stuck, regardless of energy cost or price. Finally, the response of $H_1$ to changes in total ex post operation cost could be excessive under our example, and is, at least, likely to be excessive under the Cobb-Douglas and log-linear utility specifications.

Figure 11 tells another interesting story, namely, what happens to ex ante energy consumption over the lifetime of the infrastructure, in response to ex ante, fully and rationally expected increases in expected retrofit costs in period 2. Here also two effects on energy consumption go in different directions. The ex post effect is to increase energy consumption as energy is used in more states ex post when $E_y$ increases. The ex ante effect implies that the initial energy intensity of the infrastructure is reduced, in response to the increase in (ex ante) expected costs in period 2. The most interesting result here is that, when variances on costs are high, the latter effect may dominate as the former effect is small (good substitution possibilities ex post imply that expected ex post costs increase by little when $E_y$ rises).

**Figure 11: Expected per-period energy consumption over the project’s lifetime, as function of expected retrofit cost in period 2, different utility functions, independent costs**

Figure 12 below describes effects of different degrees of correlation between energy and retrofit costs, on overall energy consumption over the project’s lifetime. Again, only relative figures are meaningful when the correlation coefficient changes. Higher cost correlation leads, in almost all cases, to less ex ante energy consumption in particular as energy intensity, $H_1$, drops (as already seen from figure 6), and as the probability of energy use, while seen to increase from figure 2, increases only slightly. This latter effect may however in some cases be sufficiently strong for energy consumption to increase overall in some cases (as under the Cobb-Douglas utility function, when correlation is already high).
Overall, some striking conclusions, highly relevant for energy and climate policy, can be drawn from these simulations. Given that the expectation of unconditional energy cost $E(q(2))$ is set at a given level (in our case then, equal to 2), the conditional or actual expected energy cost related to the infrastructure in period 2 is lower in all cases. The difference is greater when variances (of both $q$ and $y$) are larger. This is because the decision maker alternatively, and optimally, exercises the better of three options ex post, energy use, retrofit, or closedown. Energy costs are avoided in states of the world where such costs are high, but also when they are low but retrofit costs are even lower. These (cost-avoidance) effects are stronger, the more variable energy and retrofit costs are (for given unconditional expectations).

Focusing on energy costs, (conditional) expected energy costs are reduced when costs become more variable (for given unconditional expectations), due to three separate reasons. It then becomes more likely that a) a given retrofit cost is lower, and that b) a given utility value of continued operation is lower than actual realized energy cost. Besides, c) a more variable retrofit cost increases the likelihood that the retrofit cost (with given expectation) is lower than any given energy cost. All these factors tend to ameliorate the overall effect of the initial “tying up” of energy costs associated with a given infrastructure.

**Figure 12: Expected per-period energy consumption over the project’s lifetime, for different levels of uncertainty, and function of degree of cost correlation**

The figures illustrate a further feature of the theoretical analysis, namely that when expected (unconditional) retrofit cost is greater than expected energy cost (for given $\text{var}(q)$), expected conditional or actual energy cost is greater than expected retrofit costs. This is because when a given expected cost is high, the respective alternative tends to be exercised in fewer cases.
A few cautionary notes must be recognized when interpreting these simulations. In particular, the model has the (perhaps unrealistic) feature that when the closedown option is exercised, there is no energy consumption or any emissions. The same holds when the retrofit option is exercised. This assumption could however, without much loss of generality, be weakened by assuming a certain (minimum) level of energy use and emissions in this case.

4. Summary and Final Comments

The main purpose of this paper has been to analyze the implications of two interacting sets of decisions concerning infrastructure investments with long lifetimes and that commit society to potentially high levels of energy use, and carbon emissions, for a long future period after the investments have been made. One of these decisions is the (basic and initial) energy intensity of the infrastructure investment. In section 2 above we studied factors behind such investments. With particular reference to the class of so-called constant relative risk aversion (CRRA) utility functions for characterizing the relationship between energy intensity and the public’s utility from the infrastructure, we noted that the energy intensity of the initially established infrastructure can respond more or less strongly to a change in its future expected cost of operation (considering different infrastructure types that all have the same initial investment cost). We discussed three classes of cases: a) one where initial energy intensity is increased more than proportionately to a given relative increase in ex post operation costs (when the utility function takes a Cobb-Douglas form); b) the case where energy intensity increases proportionately to the reduction in overall cost (the logarithmic case); and c) a set of cases where it increases less than proportionately (the exponential case). We indicated that case c), which implies that the price elasticity of demand for energy through infrastructure energy intensity is less than unity in absolute value, seems as the most empirically reasonable of the three cases. We indicated that the implication of a change in ex post costs for initial energy intensity, and as a result overall expected lifetime energy costs and carbon emissions, could be substantial.

A second main aim of the paper has been to study impacts of two types of “ex post” policy interventions, that may be applied after an infrastructure investment has been sunk. The first is a “retrofit” of the infrastructure (by making an additional later investment that then removes the energy demand and/or emissions while retaining its utility value to the public). The second policy option is simply to close the infrastructure facility down (which is a more drastic alternative, as the utility value of the infrastructure is then is lost, together with energy use and emissions). Most of the focus here has been on the retrofit alternative, and on its ability to reduce subsequent (energy and environmental) costs, and its effect back on energy intensity of the initial infrastructure.

These objectives of the paper must be viewed against the 5 types of market failure, discussed in section 2 above, that can result in inefficient infrastructure choices. Roughly, these explanations can be classified into two groups. The first group is related to insufficient or faulty general climate-related or energy policies (including insufficient emissions pricing and technology support); while the second group is related to inefficiencies and incompleteness related to expectations about future policies. Points B and D fall largely into the former category, while points A and C fall mainly into the latter. Point E might conceivably fall into either category.
It is easy to understand why policies of individual governments hosting infrastructure projects are often faulty: namely, the basic lack of incentives of governments to address the problems of mitigation, at least in the absence of a comprehensive and binding agreement requiring the governments to behave in an optimal fashion. The second group of explanations has a more diverse set of explanations, which are however all related to either policy incompleteness or “irrationality” (or “behaviorism”) in the policy process.

We now discuss the results from our simulations, presented in section 3 above and in appendix B, in light of these 5 points.

A) The initially expected distribution of energy costs, relevant for making current infrastructure decisions, is down-shifted relative to the true (correct) distribution that will be realized in period 2. This causes infrastructure decisions to be incorrect, and chosen with too high energy intensity. Figure 7 in section 3.3 then represents the degree of distortion (as the scaling of the initial infrastructure, which is proportional to final lifetime energy demand for the infrastructure, is in this case not optimal and generally excessive). Ex post decisions for given, already established, infrastructure are however assumed to be socially optimal. The energy consumption choice, in response to an increase in “true” energy cost which is not anticipated nor ignored, is then valid only for period 2. It is represented by simulations in figure 1 (figure 1.1 in appendix B for correlated costs). In our examples, the scope for ex post avoidance of costs could be substantial as expected energy costs increase (most for the Cobb-Douglas specification and least for the exponential specification). The fully optimal response, involving also optimal ex ante infrastructure choice, is characterized by figure 10 and is generally greater (in some cases far greater). To provide an example, assume that the correct value of $E_Q$ is 4, while the assumed value is 2. Focusing now on the log and exponential specifications, we find that the chosen energy intensity of infrastructure is in the range 30-50 percent higher than the optimal level in this case. This is then also the degree of excessive carbon emissions in this case.

B) Energy (including environmental) costs facing the policy maker are lower than globally optimal costs. The distribution of energy costs facing the policy maker is however correctly anticipated. The energy cost distribution applied by the policy maker is then down-shifted relative to the “optimal” energy cost distribution (the charge to the project, for combined energy and environmental costs, is lower than the global cost); and project energy costs are not responsive to increases in true environmental costs. In this case, there is (in the limit with no increase in the local energy cost in response to a “true” cost increase) no response whatsoever to an increase in energy/environmental costs, neither ex ante nor ex post. The degree of distortion to lifetime energy consumption can then be found directly from figure 10, in section 3.4. This is the maximal distortion to this variable found here; both the initial energy intensity, and the ex post choice of energy consumption versus retrofit, are distorted upward for a large effect. The distortive effect can be read out of figure 10 by comparing values at “correct” versus “accounted for” values of $E_Q$. Considering the same example as under case A (where the true expected energy cost is 4, while the expected cost facing policy makers is 2; and again considering the log and exponential specifications), the degree of excess consumption of fossil fuels is now more dramatic, between 2 and 3.5 times the optimal level. This reflects the inefficient choices at two levels, namely both the initial investment level, and the ex post level at which the investment is applied.
C) *Expectations about future retrofit costs are incorrect at the stage of infrastructure investment.* Such mistakes could go both ways (retrofit costs could be either over- or under-estimated), and thus lead to either excessive or inefficiently low overall energy consumption. Assume first that true retrofit costs are under-valued. Referring to our simulations, we must then consider the infrastructure decision which is biased by this distortion. Figure 8 (and figure 3.2 in the appendix, for the correlated cost case) provides the clue to answering this question. In particular, under the log-linear or exponential utility cases, with an under-valuation of the ex ante expected retrofit cost, set at 2 instead of a correct value of 3, infrastructure is too energy-intensive, but not dramatically so: energy intensity is excessive by about 10 percent or less in all examples. With an excessive assessment of the retrofit costs, the mistake goes in the opposite direction (so that infrastructure has too low energy intensity), but again only slightly.

D) *Future retrofit costs are correctly anticipated, but are higher than socially optimal costs.* The idea here is that future retrofit costs are endogenous and affected e.g. by investments made in technology to reduce these costs; and such investments may be under-provided. In this case we need to consider lifetime energy consumption (both initial investments, and ex post energy use), and its response to changes in $E_y$, described in figure 11 (and figure 4.2 in the appendix). Two factors then affect overall energy use, and in different directions. First, the higher than optimal expected future cost (implied by the higher retrofit cost) leads to an inefficiently low ex ante energy intensity of the initial infrastructure. Secondly, the ex post probability of energy use is excessive, since retrofit is used in too few cases. We find, from figure 11, that the balance between these two factors depends heavily both on the degree of cost uncertainty, and on the utility function specification. We first note that total energy consumption is rising with the level of the retrofit cost, in all cases of the simulations under the log and exponential utility specifications. We also note that when cost uncertainty is low, the ex post effect, representing the ex post probability of continued energy use, always dominates, and overall expected energy use increases under all utility function specifications. With greater variances on the cost components, however, the overall probability of energy use changes much less; and the two factors more or less cancel out in the simulations.

E) *The policy-relevant value of $T$, $T_1$, is below its optimal value, $T_0*. Such effects are less directly represented by our simulations. It may be represented via a too low weight to period 2 and its ensuing costs, when making the infrastructure decision. The main effect is that the energy intensity of the initial infrastructure, and in consequence energy consumption, is excessive. There is however less reason why the ex post retrofit decision should be distorted.

Policy incompleteness (applying to point E above) could arise as an issue when policies are based on ad hoc rules that are outcomes not of deliberate optimisation, but instead a simplified process that may lead to systematic biases in a climate context. One such case is when the rate of discount for evaluation of public projects with climate impacts is determined administratively for large classes of projects (typically at a high rate), and not aligned with optimality rules relevant for (long-run) climate-related projects. Another case of policy incompleteness is when the returns to public projects are accounted for only over a limited horizon (say, 20 years), or the project is not based on explicit cost-benefit calculations.
Considering implications for initial infrastructure design of problems related to categories A-E above, we have seen, supported by our simulations, that all categories except D overwhelmingly tend to illustrate cases where energy consumption is excessive over the lifetime of an established infrastructure. In cases A-B, this occurs in two complementary ways: through excessive energy intensity of the initial infrastructure; and through excessive ex post “business-as-usual” operation of the investment in period 2. Under case C, initial energy intensity is excessive, while “business-as-usual” operation is not affected. On balance expected energy use is excessive. The fourth category (D) is different in that it tends to make energy intensity of the initial investment lower than optimal; while the retrofit option is used ex post in too few cases. Our simulations indicate that these two factors may balance out for overall energy consumption; or (when variances on energy and retrofit costs are low) lead to excessive overall energy consumption as the latter factor dominates. The fifth category, E, is similar to C as the ex ante infrastructure is biased in the direction of too high energy intensity, while the ex post decision is not generally biased.

Arguably, cases A-B are keys to understanding the implications of (perceived and actually realized) energy prices on the path of future energy consumption following from particular infrastructure investment projects. Table 1 below sums up some main results from our simulations in section 3, which are done under the assumption of uncorrelated energy and retrofit costs. We present two sets of figures. A) Those based on “policy error ex ante only”, which can be found from figure 7. These are excessive rates of fossil energy consumption over the lifetime of the infrastructure, that result from the investment decision itself being wrong; but where adjustments to correct realized energy price are made ex post. The only source if excessive energy consumption is thus that the infrastructure investment is based on wrong expectations about future energy prices.

Disregarding the Cobb-Douglas case (which seems less realistic than other cases), our simulations show that when the actual energy/environmental cost has expectation 3 (while the ex ante anticipated expectation is 2), the degree of excessive energy consumption due to “wrong” infrastructure is roughly between 20 and 35 percent; when the correct expectation is 4, this degree of excessive energy consumption is 30 and 60 percent.

When there is in addition a policy error ex post, so that the correct energy/environmental price is not implemented but instead the lower price is used, the overall error is far greater. Focusing also now on the log and exponential cases, the overall policy error (taking the compounded effect of the excessively energy intensive infrastructure, and the excessive energy use ex post) is now from about 65 percent to more than 100 percent of the efficient level when the correct Eq level is 3 (while the energy price actually applied is 2). When correct Eq = 4, this error is dramatically higher, most so for the case of low variances.26

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26 This particular result has a natural explanation, namely, that for these simulations we assume throughout that Ey = 3. Since it here makes a big difference for realized energy consumption whether Eq is smaller or greater than Ey, there is a big jump in realized energy consumption as Eq goes from a level below to a level above Ey. This jump could be unrealistically high in particular when variances are small.
Table 1: Relative policy error, in the form of excessive carbon emissions over the lifetime of the infrastructure, in percent as a result of incorrect a) ex ante expected energy/environmental cost (=2); and b) ex post expected realized energy/environmental cost (=2), for different correct Eq, based on simulations in figures 7 and 10.

<table>
<thead>
<tr>
<th>Type of policy error</th>
<th>Utility function specification</th>
<th>Low variances</th>
<th>Medium variances</th>
<th>High variances</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eq=2.5</td>
<td>Eq=3</td>
<td>Eq=4</td>
<td>Eq=2.5</td>
</tr>
<tr>
<td>Policy error ex ante only</td>
<td>Cobb-Douglas</td>
<td>39 75 124</td>
<td>40 79 146</td>
<td>40 81 158</td>
</tr>
<tr>
<td></td>
<td>Logarithmic</td>
<td>18 32 50</td>
<td>18 34 57</td>
<td>18 35 61</td>
</tr>
<tr>
<td></td>
<td>Exponential</td>
<td>12 21 31</td>
<td>12 21 35</td>
<td>12 22 37</td>
</tr>
<tr>
<td>Policy error both ex ante and ex post</td>
<td>Cobb-Douglas</td>
<td>68 177 693</td>
<td>64 154 446</td>
<td>62 147 403</td>
</tr>
<tr>
<td></td>
<td>Logarithmic</td>
<td>42 109 430</td>
<td>39 90 248</td>
<td>37 84 213</td>
</tr>
<tr>
<td></td>
<td>Exponential</td>
<td>34 91 363</td>
<td>31 72 199</td>
<td>30 66 168</td>
</tr>
</tbody>
</table>

A main objective of this paper has been to study impacts of options to retrofit an established infrastructure on total expected costs, expected energy (including environmental) costs, and initial energy intensity of infrastructure. Our simulations provide a rich set of examples of how overall realized (conditional) expected costs vary in response to variations of certain key parameters (expectations and variances), and given that ex post energy and retrofit costs are both log-normally distributed (and independent). They indicate that some fraction of expected future energy use related to infrastructure can always be avoided by optimally exercising either the retrofit or close-down option at a later stage, given that exercising such options is ex ante optimal. This degree of cost avoidance can be large, under plausible assumptions; and larger when there is more uncertainty about energy and retrofit costs. In certain parametric cases more than half of the (ex ante expected) potential energy consumption is avoided through optimally exercising the retrofit alternative ex post. Expected total (energy plus retrofit) costs are then reduced, in some cases substantially.

Note that with a long expected time from infrastructure investment to availability of a relevant alternative option (retrofit, or closedown), and when the decision maker discounts too heavily (excessively), the options will also be discounted too heavily and given too little weight in the infrastructure decision problem. This factor would then serve as a partial counterweight to those emphasized here, that tend to reduce energy (and climate) cost below socially efficient levels, and lead to too energy intensive infrastructure choices.

Several limitations of our analysis must be pointed out. First, it is based on ex ante distributions of energy and retrofit costs that are both log-normal and known. Secondly, we assume only two periods, “the present”, and “the future”, which allows for only one decision point beyond that of infrastructure investment (at the start of period 2). Our choice of assumptions here was guided by a concern for generality in the distributional assumptions, while still permitting a tractable analysis. An extension of the current framework to three or more periods would make the analysis less tractable, but ought to be pursued in follow-up work. Relevant costs (of energy, emissions, and related to the retrofit technology), as well as benefits (the current utility value of the infrastructure technology), all in reality evolve
continuously through time making the two-period framework less accurate. An obvious development would then be to assume that retrofits could be carried out at several points of time; and with separate developments for energy and retrofit costs. Such extensions are being considered in a companion paper to the current one, Framstad and Strand (2010), where energy and environmental costs are assumed to evolve continuously over time; in other respects however assumptions are much simpler in that paper, in particular, a fixed retrofit cost is assumed.27

Our analysis, both theoretical and for simulations, assumes that the scaling of the initial infrastructure does not interact with the ex post decision to retrofit. This follows from our basic assumption that retrofit costs are proportional to energy consumption. When comparing overall retrofit costs with overall energy costs ex post for a given project, project size then does not matter; only energy and retrofit prices matter. This is, admittedly, a special case, which could be generalized. With such a generalization, the ratio of overall retrofit costs to overall energy costs would likely be a decreasing function of project size (unit retrofit costs would be reduced for larger projects). Additional effects would then arise. In particular, larger ex ante projects would be favoured more as these would, effectively, permit more cost avoidance later on (through more frequent retrofitting). It is less clear how overall energy consumption over the project’s lifetime would be affected; the initial scale-up of projects, and the subsequent more frequent retrofitting, would work in opposite directions.

Our choice of utility functions for simulations might need some comment. One of our applied specifications, the Cobb-Douglas formulation, can be argued to yield unrealistic results as the response of energy intensity in infrastructure, to changes in expected lifetime project costs, is too great (with an elasticity for energy intensity in infrastructure to ex post cost which is greater than one in absolute value). This formulation is thus likely to be less credible than the other two utility function formulations, in yielding respectively a unitary (the log formulation) and less than unitary price elasticity (the exponential formulation).

A further extension would be to consider (partial) retrofits where neither fossil energy use nor carbon emissions are reduced to zero, which may be more cost efficient in some cases. Also, in our model, upon a possible infrastructure closedown, “nothing happens”. A more satisfactory analysis would involve a replacement infrastructure taking over the flow of services lost by closedown; which would require specific assumptions about costs (of new, future, infrastructure investment) and benefits (flowing from the new, instead of the old, infrastructure). We seek to pursue such extensions in future work.

27 A basic result here is that continuous development of costs produces an “option value” of waiting which serves to delay the retrofit decision; this lowers expected total costs but increases environmental costs.
References


Appendix A: Further Analytical Results


We will now study some implications of changes in the distribution of energy costs in period 2. Throughout appendix A we consider a general utility function specification, and general distribution functions \(F\) and \(G\). The only main limitation is that we here focus on the simplified case where \(q\) and \(y\) are independent.\(^{28}\) Consider a downward shift \(\alpha\) in the distribution of energy costs, that leaves all other parameters unchanged. Set \(F_q = F\) and \(F_y = G\). Call the new distribution \(F(q) = F(q+\alpha)\) (so that \(F\) is shifted up relative to \(F\) by a constant amount \(\alpha\) for any given \(q\)). This is formally the same as the entire distribution of \(q\) being shifted down, but (invoking the assumption of independence between the two cost variables) retaining the distribution function \(F\). Energy costs, in consequence, fall on average. In particular, since the distribution \(G\) is unaltered, the following new definition of \(E[CH(2)]\) then applies:

\[
E[CH(2)] = \left( \int_{q=0}^{y^*} \left[ 1 - G(q) \right] f(q + \alpha) q dq \right) H_1
\]

\[
E[CR(2)] = \left( \int_{y=0}^{y^*} \left[ 1 - F(y + \alpha) \right] g(y) y dy \right) H_1
\]

Differentiating the expressions for \(P(1)\), \(P(2)\), \(E[CH(2)]\) and \(E[CR(2)]\) with respect to \(\alpha\) then yields (assuming that \(P(3)\) is not “significantly” altered by any resulting change in \(y^*\)):\(^{29}\)

\[
\frac{dP(1)}{d\alpha} = \int_{q=0}^{y^*} \left[ 1 - G(q) \right] f'(q) dq + \left[ (1 - G(y^*)) \right] f(y^*) \frac{dy^*}{d\alpha}.
\]

\[
\frac{dP(2)}{d\alpha} = - \int_{y=0}^{y^*} f(y) g(y) dy + \left[ (1 - F(y^*)) \right] g(y^*) \frac{dy^*}{d\alpha}
\]

Recall the definition \(Ec(2) = E[C(2)]/H_1\) as the ex post realized cost (per time unit in period 2) per established energy unit. Define now also \(Ech(2) + E[CH(2)]/H_1\), and \(Ecr(2) + E[CR(2)]/H_1\). We then derive the following general expressions:

\[
\frac{dE[CH(2)]}{d\alpha} = \frac{dEch(2)}{d\alpha} \frac{1}{H_1} + \frac{E[CH(2)]}{H_1} \frac{dH_1}{d\alpha}
\]

\[
\frac{dE[CR(2)]}{d\alpha} = \frac{dEcr(2)}{d\alpha} \frac{1}{H_1} + \frac{E[CR(2)]}{H_1} \frac{dH_1}{d\alpha}
\]

\(^{28}\) A similar comparative-static analysis in the more general case, with dependent distribution functions, turns out not to yield tractable and easy to interpret expressions. For that reason we are here focusing on the case of independent distributions.

\(^{29}\) This requires, under the quadratic utility specification in particular, that the coefficient \(b\) in (14a) is small, and that \(H_1\) in response changes “relatively much” compared to \(y^*\).
$P(1)$ here increases (as $f'$ is positive for low $G$ values): the probability of “business as usual” increases. This is intuitive: when energy costs fall, the likelihood that the (business-as-usual) energy cost option is exercised in period 2 increases, “everything else equal”. The probability that the retrofit option is exercised in period 2 drops unambiguously. The increase in the former is greater so that $P(1)+P(2)$ increases (i.e. the closedown option, is exercised in fewer cases). Ignoring first effects via changes in $H_i$, we find that the effect of a shift in $\alpha$ on unit energy costs (represented by the first term on the right-hand side of (A3)) is ambiguous. Two factors go in different directions: a greater $P(1)$ implies that energy costs are incurred in more states, leading such costs to increase. On the other hand, unit energy costs drop for any given state, which tends to reduce costs. The effect on expected unit retrofit costs is however unambiguously negative. This is intuitive: the only thing that happens to retrofit costs is that such costs are applied in fewer states, thus reducing overall expected retrofit costs.

Note here that while $dP(1)/d\alpha$ is positive, $dE[CH(2)]/d\alpha$ can be either positive or negative, depending on the distributions. Consider now the CRRA case (with relative risk aversion coefficient = $\rho$). We then find

\[
\frac{dH_i}{d\alpha} = \frac{1}{\rho} \frac{TH_i}{q+TEc(2)} \frac{dEc(2)}{d\alpha}
\]

\[
\frac{dy^*}{d\alpha} = \frac{Ty^*}{q+TEc(2)} \frac{dEc(2)}{d\alpha}
\]

Since (as generally found below) $dEc(2)/d\alpha$ is generally negative, the change in $H_i$ is positive, as expected: a lower overall expected cost in period 2 makes it attractive to set a higher energy intensity for the initial infrastructure. But the cut-off point $y^*$ for ex post costs (beyond which closedown will be selected) is reduced. This serves to somewhat increase the frequency with which the closedown option will be exercised ex post. Note that $y^*$ falls in $H_i$, rather generally and more specifically for our utility function specification (36).

Differentiating $Ech(2)$ and $Ecr(2)$ we find:

\[
\frac{dEch(2)}{d\alpha} = \left( \int_{q=0}^{y^*} (1-G(q))f'(q)qdq \right) + [1-G(y^*)]f(y^*)y^* \frac{Ty^*}{q+TEc(2)} \frac{dEc(2)}{d\alpha}
\]

\[
\frac{dEcr(2)}{d\alpha} = \left( \int_{q=0}^{y^*} f(q)g(q)qdq \right) + [1-F(y^*)]g(y^*)y^* \frac{Ty^*}{q+TEc(2)} \frac{dEc(2)}{d\alpha}
\]

(A7)-(A8) solve simultaneously for these two derivatives, noting that $dEc(2) = dEch(2) + dEcr(2)$. In considering this system, we find that meaningful solutions (where both derivatives have the same sign as the respective first expressions on the right-hand sides) requires the following condition to hold:
This condition puts bounds on \( f(y^*) \) and \( g(y^*) \) (densities of the respective distribution functions in the neighbourhood of \( y^* \)): these cannot be too large at an equilibrium solution.

In general, we cannot unambiguously determine the sign of \( dEch(2)/da \), but it must have the same sign as the first term on the right-hand side of (A7). This has an intuitive interpretation: ex ante unit energy costs may go down if the cost shift factor \( (\alpha) \) dominates; but it may go up if the “probability factor” (representing a higher likelihood that “business as usual” energy use will be chosen in period 2) dominates. \( dEcr(2)/da \) must, by contrast, be negative: the only thing that happens to effective retrofit costs in this case is the likelihood of a retrofit is reduced, implying that effectively applied costs are reduced.

\( dEc(2)/da \) must in general be negative (which is obvious as unit costs are generally reduced).

What happens to overall costs as a function of \( \alpha \), when also the effect on \( H_1 \) is accounted for? The effect of the shift in the cost distribution on overall expected cost, \( E[C(2)] \), is given as the sum of the terms from (A3) and (A4). Assuming still CRRA, we find

\[
(A10) \quad \frac{dE[C(2)]}{d\alpha} = \frac{dE[c(2)]}{d\alpha} H_1 \left( 1 - \frac{TEc(2)}{\rho q + TEc(2)} \right).
\]

Here, as noted, \( dE[c(2)]/d\alpha < 0 \) (from the system (A7)–(A8)). Thus, \( dE[C(2)] < 0 \) as long as the last parenthesis is positive. This requires that

\[
(A11) \quad \rho > \frac{TEc(2)}{q + TEc(2)}.
\]

Consider here the case where \( \rho = 1 \), and the utility function logarithmic. Then \( dE[C(2)] < 0 \) insofar as \( q > 0 \), and more so the larger \( q \) is in relation to \( TEc(2) \) (in other words, when first-period costs are a larger fraction of expected overall operating costs over the lifetime of the facility). In particular (and as discussed in section 2.2), when \( q = 0 \) (first-period costs can be ignored), a negative shift in the distribution of ex post energy costs leads to an decrease (increase) in total expected overall costs when also scaling the initial energy intensity is considered, given \( \rho > (\leq) 1 \). Thus in particular when \( \rho < 1 \), any initial cost reduction from the positive shift to \( \alpha \) is more than eliminated through a higher choice of energy intensity, associated with the initial infrastructure investment.

Consider next the effects on carbon emissions from a shift in \( \alpha \). The expression for expected ex ante carbon emissions, denoted \( E \), is given by

\[
(A12) \quad E = H_1 [1 + TP(1)].
\]

Note here in particular that carbon emissions equal \( H_1 \) whenever the “business as usual” alternative is being pursued; this alternative is pursued with probability 1 in period 1, and by ex ante probability \( P(I) \) in period 2. Thus:
\( \frac{dE}{d\alpha} = [1 + TP(1)] \frac{dH_1}{d\alpha} + H_1T \frac{dP(1)}{d\alpha} \)

\( dP(1)/d\alpha \) is here positive, and so is \( dH_1/d\alpha \). Thus carbon emissions increase, for two separate reasons; first, energy intensity of the infrastructure, \( H_1 \), increases; and secondly, the probability of the ex post business as usual alternative (with normal energy consumption), \( P(1) \), increases. Note however that \( P(1) \) increases less as a result of \( y^* \) being reduced in response to the higher \( H_1 \).

More specifically in the CRRA case we have the following expression:

\[
\frac{dE}{d\alpha} = TH_1 \int_{q=0}^{y^*} [1 - G(q)] f(q) dq - \left\{ \frac{1}{P} \left[ (1 + TP(1)) - T [1 - G(y^*)] g(y^*) y^* \right] \right\} TH_1 \int_{q=0}^{y^*} \frac{dEc(2)}{q + TEc(2)} d\alpha
\]

**A2. Shifts in the Retrofit Cost Distribution with Cost Independence**

This subsection derives some comparative-static results regarding impacts of changes in costs of retrofitting in period 2. Again we focus on the case of independent energy and retrofit costs (\( q \) and \( y \)). We study impacts on outcomes, from both marginal changes in retrofit costs, and from the retrofit option being at all available.

\( E[CH(2)] = \left( \int_{q=0}^{y^*} [1 - G(q + \beta)] f(q) dq \right) H_1 \)

\( E[CR(2)] = \left( \int_{y=0}^{y^*} [1 - F(y)] g(y + \beta) dy \right) H_1 \)

Differentiating the expressions for \( P(1), P(2), E[CH(2)] \) and \( E[CR(2)] \) with respect to \( \beta \) then yields

\[
\frac{dP(1)}{d\beta} = - \int_{y=0}^{y^*} f(y) g(y) dy + [1 - G(y^*)] f(y^*) \frac{dy^*}{d\beta}
\]

\[
\frac{dP(2)}{d\beta} = \int_{q=0}^{y^*} [1 - F(q)] g'(q) dq + [1 - F(y^*)] g(y^*) \frac{dy^*}{d\beta}.
\]

\[
\frac{dE[CH(2)]}{d\beta} = - \left( \int_{q=0}^{y^*} f(y) g(y) dy \right) H_1 + \frac{E[CH(2)]}{H_1} \frac{dH_1}{d\beta}
\]

\[
\frac{dE[CR(2)]}{d\beta} = \left( \int_{q=0}^{y^*} (1 - F(q)) g'(q) dq \right) H_1 + \frac{E[CR(2)]}{H_1} \frac{dH_1}{d\beta}
\]
Interpretations are similar to the case of energy cost changes. When $\beta$ increases, the distribution of retrofit costs (in analogous fashion to the energy cost distribution, in section A1) shifts downward, and average retrofit costs fall. The probability of “business-as-usual” energy consumption ($P(1)$) then decreases unambiguously, while the probability of retrofit ($P(2)$) increases unambiguously. The increase in the latter is also now in general greater, so that $P(1)+P(2)$ increases. Thus expected energy intensity of the infrastructure falls unambiguously (as represented by the integral on the right-hand side of (A17)), while the change in expected retrofit cost per established energy unit is ambiguous (the integral on the right-hand side of (A18)). Their sum, $EC(2)$, falls unambiguously.

Consider now, in the same way as in section A1 above, changes in energy intensity of the infrastructure ($H_1$), and energy use and carbon emissions. Focusing again on the CRRA case, the expression for effects on $H_1$ is still given by (A5) except that $\alpha$ is replaced by $\beta$. This means that the effect on $H_1$ is quite similar to that in section 3, for a shift in energy costs. The expressions for the effects on $Ech(2)$ and $Ecr(2)$ are also similar and given by

\[
\frac{dEch(2)}{d\beta} = -\int_{\theta=0}^{\theta^*} f(y)g(y)dy + [1 - G(y^*)]f(y^*)y^* \frac{Ty^*}{q+TEc(2)} \frac{dEc(2)}{d\beta}
\]

\[
\frac{dEcr(2)}{d\beta} = \int_{\theta=0}^{\theta^*} (1-F(q))g'(q)qdq + [1 - F(y^*)]g(y^*)y^* \frac{Ty^*}{q+TEc(2)} \frac{dEc(2)}{d\beta}
\]

Moreover, a similar condition to (A9) must hold for stability of the system (42)-(43) to give a meaningful simultaneous solution to $Ech(2)$ and $Ecr(2)$ and thus $Ec(2)$.

The main difference from section 3 is that the effect on carbon emissions of a parameter shift now is different. The effect on $E$ is still given by (A13) (only replacing $\alpha$ by $\beta$). The main difference arises as $P(1)$ is now affected differently. The expression for the effect on $E$ takes the form

\[
\frac{dE}{d\beta} = -TH_1 \int_{\theta=0}^{\theta^*} f(y)g(y)dy - \left\{ \frac{1}{\rho} [(1+TP(1)] - T[1 - G(y^*)]g(y^*)y^* \right\} \frac{TH_1}{q+TEc(2)} \frac{dEc(2)}{d\beta}
\]

Here the first main expression (indicating the main effect on probability of business as usual as this alternative now is replaced by the lower-cost retrofit alternative) is now negative, while the second (which mainly indicates the effect via higher initial energy intensity) is positive as in (A14). In general we cannot say which of the two terms dominates. It is however clear that when $\rho$ is greater than unity (and the response of $H_1$ to an expected change in future costs is relatively small), overall (ex ante expected) energy use is likely reduced, as the drop in ex post energy use is relatively large in this case.

Consider here an alternative case where the retrofit option is no longer available (NR denoting the “no retrofit” case)\textsuperscript{30}. We have the following probability of closedown in period 2:

\textsuperscript{30} One interpretation of such a case is that the lower bound of the retrofit distribution, $y_0$, is higher than the average total value (per unit of energy consumed) of the infrastructure in period 2.
\[ P_{NR}(3) = 1 - F(y^*), 1 - P_{NR}(3) = F(y^*). \]

In this case the probability of (energy-demanding) infrastructure operation in period 2, \( P_{NR}(1) \), is given simply by \( 1 - P_{NR}(3) \). For given \( y^* \), the probability of closedown is smaller when the retrofit option is available, \( (P(3)) \), than when it is not \( (P_{NR}(3)) \), by a factor \( (b - y^* + y_0)/b \). The probability of operation (with or without retrofit) is correspondingly greater when a retrofit option is available. The probability of energy-demanding operation is smaller with the retrofit option, by a factor \( (1 - (y^* + q_0)/2b) \).

To study how a lack of retrofit option changes the initial energy intensity of the infrastructure, \( H_1 \), assume a further simplified case with no energy cost in period 1 \( (q_1 = 0) \). The objective is then simply to compare the expected per-unit combined energy and retrofit cost in period 2 in the two cases. This cost equals \( (EC(2)/H_1)/(1 - P(3)) \) in the case where the retrofit option is included, and \( (EC_{NR}(2)/H_1)/P_{NR}(1) \) in the case where the retrofit option is not included. These are the respective expressions for average overall costs per operation time, or probability of operation in period 2. In either case this expression is to be set equal to \( U'(H_1) \), for an optimal \( H_1 \) level to be achieved.

### A3. The Value of the Closedown Option

We will now, rather briefly, consider some effects of the closedown option on the solution, initial energy intensity, and on cost variables. The closedown option is practically irrelevant when total utility per unit of energy consumed for the chosen infrastructure, \( y^* = U(H_1)/H_1 \), is so high that closedown is “almost never” used (i.e., \([1 - F_q(y^*)][1 - F_y(y^*)]\) is “very small”). This may apply to cases where the infrastructure involves a high sunk cost relative to energy consumption (such as, perhaps, for urban structures including housing and transport systems).

The expected energy and retrofit cost, and ex ante probabilities of “business-as-usual” operation and retrofit in period 2, are still given by (2)-(6). With no closedown option, \( P(1) + P(2) = 1 \). Expected ex ante utility of second-period operation is now

\[ EW(2) = \{y^* H_1 - E[CH(2)] - E[CR(2)]\}T. \]

The first-period decision problem can now be formulated as maximizing \( EW(1) \), from (8). The resulting solution for optimal energy intensity of the infrastructure is now found from the following condition:

\[ U'(H_1) = \frac{q_1 + E[CH(2)] + E[CR(2)]}{H_1} \frac{T}{1 + T}. \]

(13a) can be compared to the case with closedown, where \( H_1 \) is determined from (13). There are two main differences between (13) and (13a). First, \( E[CH(2)] \) and \( E[CR(2)] \) are greater in (13a). Secondly, \([1 - P(3)]\) in the denominator of (13) is missing in (13a). As a result, overall expected costs are greater, and the weight to second-period costs versus first-period costs is greater. Thus, when expected second-period costs “per period” are greater, this also tends to increase the overall expression on the right-hand side of (13). Overall, \( U'(H_1) \) is higher, and
Having a closedown option increases the energy intensity of the original infrastructure investment. This is intuitive: the option will be used only in states where both retrofit and energy costs are very high, and eliminates costs in these states, which in turn provides incentives to raise the infrastructure’s initial energy intensity.

A factor in relation to closedown is the curvature of the \( U \) function in particular with “significant” degrees of risk aversion. A higher \( H_1 \) then reduces \( U(H_1)/H_1 \), perhaps substantially. The threshold for ex post closedown, when this decision is compared to the higher of energy and retrofit costs, is then reduced. Closedown is exercised in more states of the world, the higher is the initial energy intensity. This is reasonable, and tends to dampen the overall energy and carbon footprint of an initially energy-intensive infrastructure.

We have assumed that, after closedown, no energy expenditure is incurred whatsoever. This is unrealistic since the closed down infrastructure will in practice need to be replaced by some alternative that is likely to be in itself energy-demanding (although presumably less so than that initially established; this would tend to follow since replacements occur in states with high energy costs in period 2).

### A4. Endogeneity of Retrofit Costs

The retrofit options available in period 2 are likely to follow at least in part from technology developments over the course of period 1, and these may in turn be influenced by R&D efforts. This section considers possible effects of such efforts. Again, the explicit focus is on cost independence which simplifies the analysis considerably.

Influencing R&D efforts with the purpose of mitigating GHGs has emerged as a core theme in the climate policy debate, from several angles. One is how an optimal climate and energy policy (in the form e.g. of emissions or energy taxes) can depend on the presence of R&D; this has been discussed e.g. by Goulder and Schneider (1999), Goulder and Mathai (2000), Bonanno et al. (2003), Greaker and Pade (2008), Acemoglu et al (2009), and Golombek, Greaker and Hoel (2010). A separate issue is that while it may be very difficult to reach an international agreement to reduce GHG emissions directly using policy instruments such as emissions taxes and caps, some analysts claim that reaching an agreement to support emissions-reducing technological progress may be easier.\(^{31}\)

Here we simply assume that the infrastructure investment decision maker may also carry out R&D activity to reduce the costs of retrofitting this particular infrastructure in period 2, and not for other units nor more generally. We assume that the entire distribution function for period 2 retrofit costs can be given a constant vertical shift (in similar fashion as in section 4 below) through additional R&D effort in period 1.\(^{32}\) This upward shift in distribution is the same as a downward cost shift, as in section 4, and was there given from (A18) in appendix A, as a result of changes in a shift parameter \( \beta \) for this distribution.

Here, consider the following modified discounted utility as viewed from period 1:

\(^{31}\) See in particular Barrett (2006, 2009).

\(^{32}\) This is of course highly unrealistic. In practice, R&D efforts will affect retrofit costs more generally, and also for other projects, and thus imply positive externality effects for the latter. This issue is not discussed fully here. For further discussion see e.g. Golombek and Hoel (2005, 2006), Golombek, Greaker and Hoel (2010).
(A23) \[ EW(1; R) = (y^* - q_i)H_1 + EW(2) - R \]

where \( EW(2) \) is given from (7), \( R \) is the first-period R&D cost, the \( F_y \) function is shifted, with shift parameter \( \beta \), and where the size of \( \beta \) is a positive function of \( R \). Focusing on the case of cost independence, we then derive the following general optimality condition with respect to \( R \):

\[
\frac{dEW(1; R)}{dR} = \left[ y^* - q_i + y^* (P(1) + P(2))T \right] \frac{dH}{d\beta} + y^* TH_1 \frac{dP(1) + dP(2)}{d\beta} - T \frac{dEC(2)}{d\beta} \left( \beta_R - 1 \right) = 0
\]

where \( \beta_R \) is the derivative of \( \beta \) with respect to \( R \). The partial derivatives in (A24) are found from (A19)-(A22), plus (11) differentiated. While (A24) looks complicated, its essence is that the total derivative of \( EW(1) \) with respect to \( R \) consists of three marginal benefit terms inside the curled bracket, classified by how model variables are affected: 1) effects via the increase in \( H \); 2) via increase in the joint probability of period 2 operation, \( P(1) + P(2) \); and 3) via operation costs (energy costs plus retrofit costs) in period 2. These three terms are traded off against, and at the optimum set equal to, the unit cost of R&D investment.

An important parameter here is \( \beta_R \). Presumably, the marginal effect of R&D costs diminishes with greater costs (as will, rather generally, be required for a unique internal optimum to exist for the problem (A24)).

Consider a second-order Taylor expansion

(A25) \[ \beta(R) = \lambda R - \mu R^2 \]

where \( \lambda \) and \( \mu \) are positive constants, so that the first- and second-order derivatives of the \( \beta \) function are given by

(A26) \[ \beta_R = \lambda - 2\mu R^2 \]

(A27) \[ \beta''(R) = \beta_{RR} = -2\mu < 0 \]

The main point here is that when the curled bracket in (A24) is large, \( \beta_R \) will be small, and \( R \) correspondingly large. A large overall positive utility effect of a given shift in the retrofit function then leads to a large optimal R&D effort \( R \).

We have assumed that this investment only affects costs for one particular infrastructure facility. More often, such R&D expenditures are likely to have effects also on costs for other projects. The marginal social benefit of R&D is then much greater than the “private” benefit for the infrastructure project sponsor. Assume that (A25) correctly represents the overall social impact of \( R \) on retrofit costs, while the private impact is only a fraction \( h < 1 \) of the

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33 A second-order condition here also needs to be fulfilled. A sufficient condition here is that \( \beta_R \) is decreasing in \( R \).

34 One way to visualize this is to consider R&D projects carried out in sequence by their likelihood of success; when only a few projects are funded these are the most promising.

35 Another way to express this effect is that there is likely to be a high degree of “technology spillovers” associated with R&D for development of new retrofit technology; see discussions of such spillovers e.g. by Golombek and Hoel (2005, 2006), Golombek, Greaker and Hoel (2010).
social impact. In this case, the marginal change in $\beta$ when $R$ changes, as perceived privately, is also a fraction $h$ of the social impact given by (A26), and thus

$$\beta'(R; h) = h(\lambda - 2\mu R) > 0.$$  

When $h$ is smaller, $R$ must be smaller to fulfil (A24). As a result, the R&D activity will be (perhaps much) lower than optimal when most of the overall returns to private R&D accrue to others. One then faces an obvious problem of policy coordination across countries, which in principle could be as serious as that for regular mitigation policy. High appropriability of rents to developers of new technology will tend to reduce this coordination problem.

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36 With this formulation, when $h$ is small, no such non-negative $R$ can be found. Then no R&D investments will be undertaken by private agents.

37 One way of securing this is strong patent laws. But this has negative side effects, in particular, the markets in which the newly developed technologies are applied will not be competitive; see e.g. Greaker and Pade (2008).
Appendix B: Additional Simulations

1: Simulated ex post probabilities of energy use and retrofit

Figure 1.1: Probabilities of energy use and retrofit as functions of unit energy cost, for positively correlated costs.

Figure 1.2: Probabilities of energy use and retrofit as functions of unit retrofit cost, for positively correlated costs.

Figures 1.3-1.4: Probabilities of energy use and retrofit under variable energy cost uncertainty, for independent and positively correlated costs (low variances)
Figures 1.5-1.6: Probabilities of energy use and retrofit under variable uncertainty (medium variances)
Figures 1.7-1.8: Probabilities of energy use and retrofit under variable uncertainty (high variances)
2: Simulations of ex post energy and retrofit costs

Figure 2.1: Ex ante expected energy/retrofit costs in period 2 as function of expected unit energy costs, for different variances, positively correlated costs

Figure 2.2: Ex ante expected energy/retrofit costs in period 2 as function of expected unit retrofit costs, for different variances, positively correlated costs
Figures 2.3-2.4: Ex ante expected energy/retrofit costs in period 2 as function of variances (low and medium variances), independent costs.
Figures 2.5-2.6: Ex ante expected energy/retrofit costs in period 2 as function of variances (low and medium variances), positively correlated costs
Figures 2.7-2.8: Ex ante expected energy/retrofit costs in period 2 as function of variances (high variances)
3: Infrastructure investments

Figure 3.1: Size of infrastructure investment as function of expected energy cost in period 2, for different utility functions, variances, positively correlated costs

Figure 3.2: Size of infrastructure investment as function of expected retrofit cost in period 2, for different utility functions, variances, positively correlated costs
Figures 3.3-3.4: Size of infrastructure investment, for different utility functions and variances in period 2 (one variance medium; and one low), independent costs
Figures 3.5-3.6: Size of infrastructure investment for different utility functions and variances in period 2 (one variance medium; and one low) positively correlated costs
Figures 3.7-3.8: Size of infrastructure investment for different utility functions and variances in period 2 (one variance high)
4: Energy consumption per period

Figure 4.1: Expected per-period energy consumption over the project’s lifetime, as function of expected energy cost in period 2, different utility functions, correlated costs

Figure 4.2: Expected per-period energy consumption over the project’s lifetime, as function of expected retrofit cost in period 2, different utility functions, correlated costs
Figures 4.3-4.4: Expected per-period energy consumption over the project’s lifetime, for different utility functions and variances in period 2 (one variance low)
Figures 4.5-4.6: Expected per-period energy consumption over the project’s lifetime, for different utility functions and variances in period 2 (one variance medium)
Figures 4.7-4.8: Expected per-period energy consumption over the project’s lifetime, for different utility functions and variances in period 2 (one variance high)